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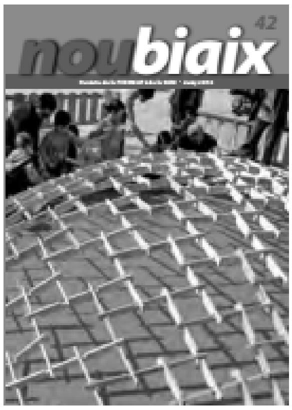

Coediten:

Federació d'Entitats per a l'Ensenyament de les Matemàtiques (FEEMCAT)
Campus de Montilivi, edifici P-IV
17071 Girona
feemcat.org

Societat Catalana de Matemàtiques (SCM), filial de l'Institut d'Estudis Catalans
Carme, 47
08001 Barcelona
scm.iec.cat
noubiaix@gmail.com
sites.google.com/site/noubiaix

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Enric Brasó

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Adolf Florensa, s/n
08028 Barcelona
Tel.: 934 035 430
Fax: 934 035 531
comercial.edicions@ub.edu
www.publicacions.ub.edu

Leonardo da Vinci’s Domes

Enric Brasó

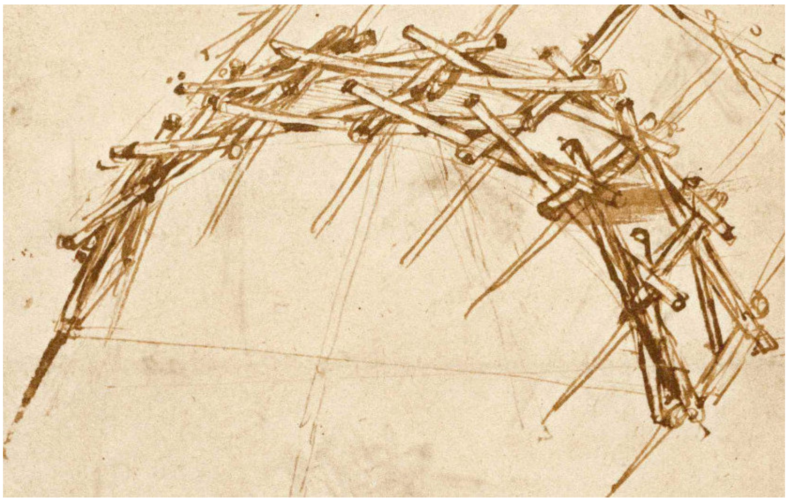
Translation into English of the original Catalan by **Milo Ramellini.**

Introduction

In this article, we present to you LeonarDome, a didactic material consisting in a number of special pieces which, using the principle of self-sustaining, allow for the construction of geometrically interesting domes. LeonarDome is available in different formats: in its greatest format, it allows for the building of structures 4 to 5 metres in diametre and 1 metre in height, which can be built in under twenty minutes by a team of 25. The construction of these domes follows different geometrical patterns, varying in complexity.

Here, we will try to explain the origins of this idea, its didactic interest, the development of the workshop and the details in design that allow for the deformation of the flat structure and its curvature. Finally, we will study different aspects of these particular tessellations and why there may be an infinite number.

The origin



Picture n.1. Self-sustaining bridge. Page 71 of Leonardo da Vinci’s *Codex Atlanticus*

More than five hundred years ago, Leonardo da Vinci, ever the genius, drew this picture of a self sustaining bridge (Picture n.1). The sticks that form the bridge simultaneously build over and support each other, allowing for the structure to stand on its own with no sustaining elements and reach a wider distance than the length of an individual piece.

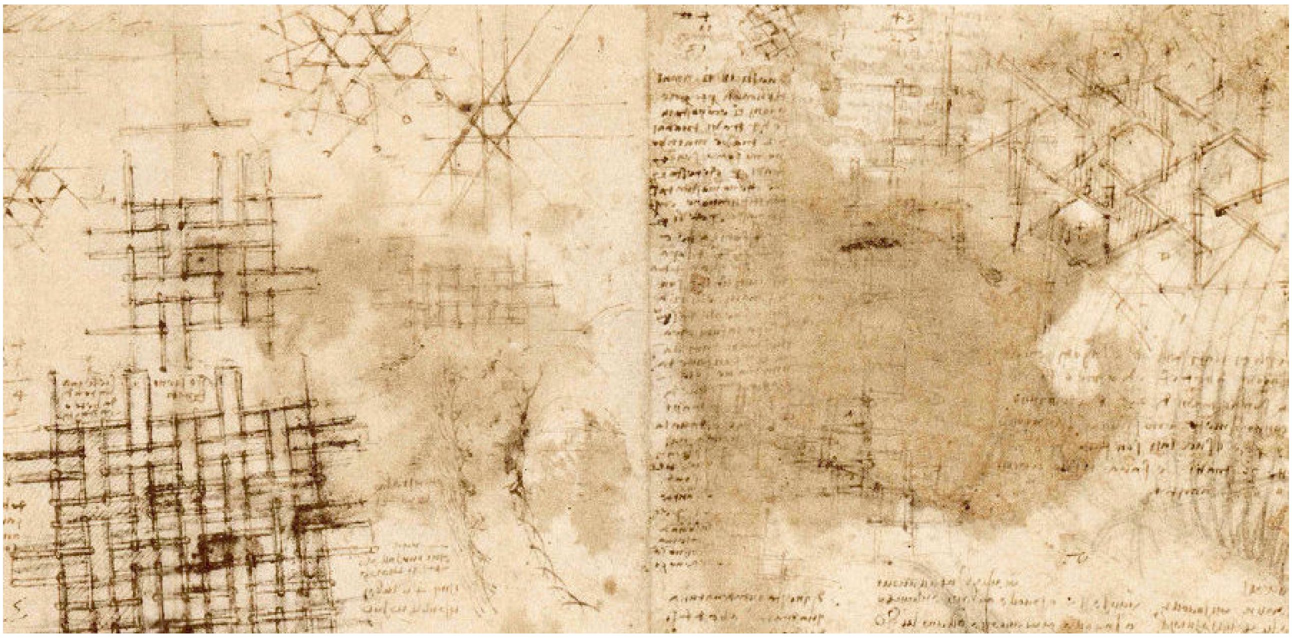
We do not know whether this idea ever saw the light in Leonardo’s time, but ever since the modern reevaluation of his writings, this bridge has been present in countless science exhibitions and museums around the world.

Leonardo’s bridge, however, builds only in one direction, covering the distance between two shores; it was only natural that someone would extend the system in order to cover a surface.

Picture n.2. First steps building a dome in two directions.

Contemporary artist Rinus Roelofs (picture n.3), whose pieces are strongly influenced by geometry, was the first one to realise and investigate on this idea[1]. Looking through Leonardo’s work, Roelofs found some sketches of what looked like tessellations of a plane using sticks in various directions (picture n.4). Rinus Roelofs develops on the artistic and geometric possibilities of its multiple variations and derivations.

Picture n.3. Rinus Roelofs, sculpturist.



Picture n.4. Page 889 of Leonardo da Vinci’s *Codex Atlanticus* and sketches.

At the Museum of Mathematics of Catalonia, where Leonardo’s Bridge has been an exhibit from the very beginning, we also realised the possibility of extending the idea of mutual overlapping in two

1 https://www.rinusroelofs.nl/structure/basi-grids/davinci-131.html

directions. We have investigated, designed and developed different simulations and prototypes focusing on the didactics of the experience. Interestingly, our own investigation and Roelofs’, which might have stressed the artistic component, have converged in size, form and dimensions.

We immediately shared the blueprint for our own design of the pieces online[2], however, it is with the beginning of production of the LeonarDome educational kit that we achieve the diffusion of these resources.

LeonarDome sticks are 50 cm long. A regular box contains 250 sticks and it includes a teaching guide (picture n.5). Furthermore, you may find more activities, workshops and support on the www.leonardome.com website.

Picture n.5. The LeonarDome Educational Kit

We have also started commercialization of a 50 piece box, more manageable in size and weight. These pieces are enough to build a small dome autonomously and is an ideal set for schools looking to set up a space for geometric experimentation.

Apart from these larger sticks, we have designed a more portable set of 15 cm long sticks in order to build domes on a table (picture n.6). These boxes also include teaching and educational resources.

Picture n.6. Tabletop LeonarDome, plastic and wooden, with and without curvature.

2 https://www.mmaca.cat

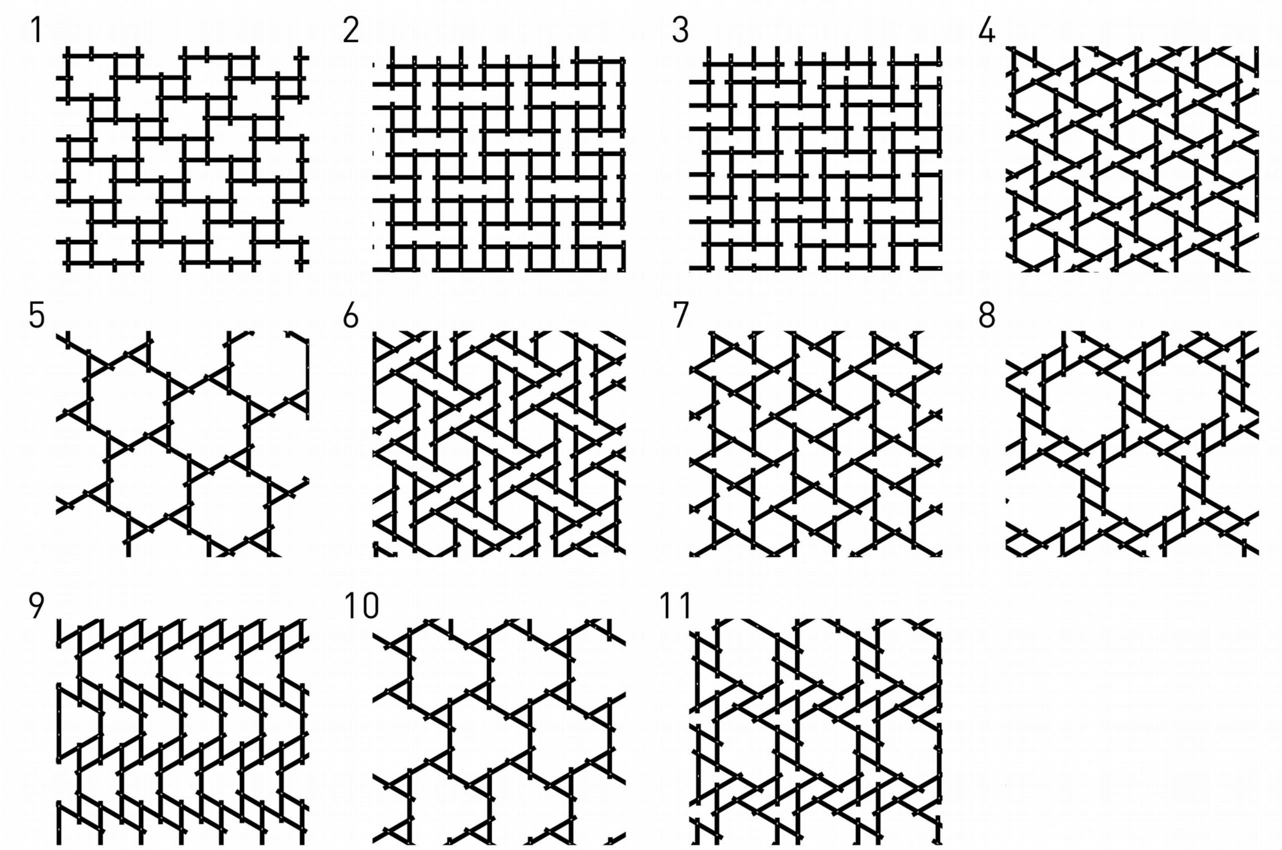
Let’s get on building

The MMACA offers a workshop around the building of Leonardo’s Domes for schools and visitors. This workshop is also available at itinerary exhibitions, schools, fairs and at demand.

This workshop is a unique experience which we have adapted for all ages: from five year old school children to university students and families. In less than 20 minutes, 20 people may build a dome 1 m tall and 4-5 meters in diametre.

First, we choose a pattern from the proposed 11 (picture n.7). Very little information is needed to begin building. Experience suggests the first three patterns with perpendicular sticks to be a good starting point. Participants are then faced with a series of challenges: they have to simultaneously and repeatedly decide on the position and orientation of each piece, following a two dimensional pattern and taking into account the overlapping system (over and under). We strongly encourage verbalizing these strategies, so that everybody can find their own way of visualizing these structures and share it.

The immediate results, the dimensions and height of the dome are a constant and collective stimulus. It’s convenient to build uniformly and coordinately. It is, therefore, a great opportunity for cooperation and mutual support. The dome, once built, is the obvious result of group work.



Picture n.7. The 11 patterns we use at MMACA.

Once the dome is built, we may carry different activities. Naturally, children will want to get inside the dome. High school students and adults may try to lift the dome coordinately in order to catch a glimpse of the dome from underneath. Once the dome is up on the air (or the hands of the participants), we can move it, deform it or even check how its flexibility gives it a resistance to earthquakes.

We have heard Maria Antònia Canals declare several times that the learning of geometry must be accompanied by manipulation and experienced through motion. These domes allow for a big part of her recommendations: to live polygons and geometric shapes from the inside.

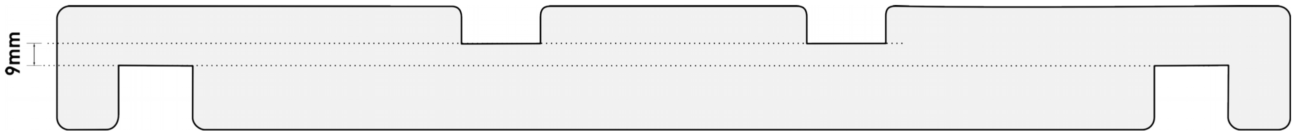
The high point of the experience, however, is the fall. If any of its structural supports should fail, the whole structure collapses like a stack of cards.

Where does this curvature come from?

An important part of the workshop is finding moments for reflection, questions and discussion. Questions may arise about Leonardo, the genesis of the idea or real applications of these domes in architecture. To us, questions about geometry are especially interesting.

- Would it be possible to continue building until we build a sphere?
- If these patterns are flat tilings, how come the domes are curved and raised?

The answer to both these questions can be found looking at the design of the sticks and realising the holes in the sticks are not leveled (picture n.8). This difference is what causes the domes to rise from a flat structure.



Picture n.8. Blueprint for the 50x5 cm stick.

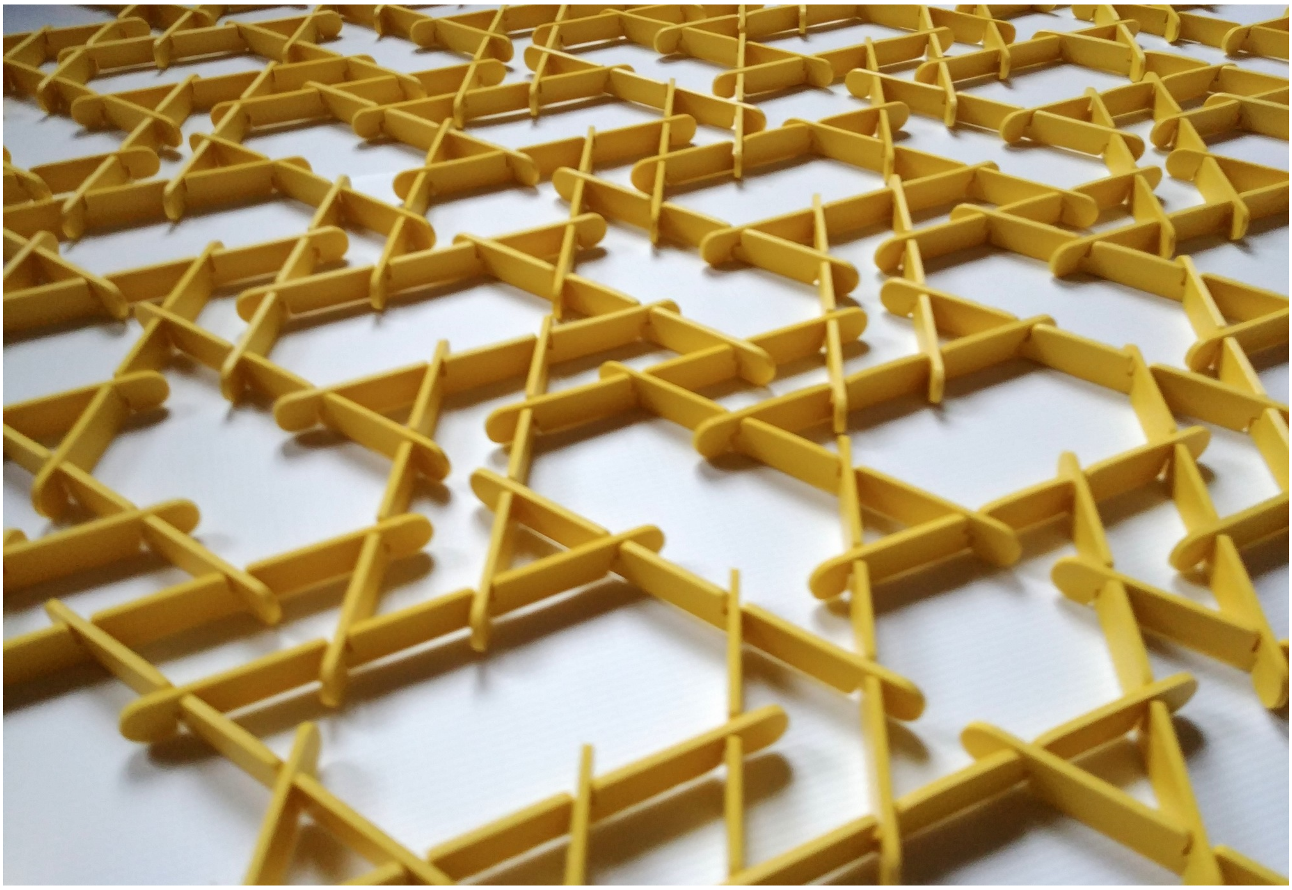
The curvature of the domes is determined by the difference in the levels of these fitting holes. If these were perfectly leveled, we would get a flat structure. We have adapted our design[3] in order to get domes big enough for a group of twenty five students.

In order for the flat surface to curve, we need flexibility, which we get from the width of the holes in the sticks. This deformed structure is far from being a polyhedron: similarly to how the roots of a tree push and separate the tiles on a floor, the vertexes on each polygon in the domes do not coincide, so that polygons don't have all elements in the same plane. The width of the fitting holes and the characteristics of each pattern dictate the growth limit of these structures. Those who have built domes quickly realize that, as the dome grows in height, the increasingly vertical position of its pieces causes gravity and friction to work less efficiently.

Therefore, construction with these patterns is not infinite. Rinus Roelofs has worked in different directions, one being the search of polyhedral patterns which can be built with Leonardo sticks. A good example if the sphere that he holds in his hands in Picture n.3.

At the MMACA, we have developed a version of the 15 cm Leonardo sticks in which the fitting holes are leveled, resulting in a structure with no curvature (picture n.9). With these sticks, instead of domes, we can create flat tessellations which can expand indefinitely. They can work as a great tool in order to investigate different patterns and their characteristics.

3 Which can be downloaded with a Creative Commons free license at www.mmaca.cat/les-cupules-de-leonardo

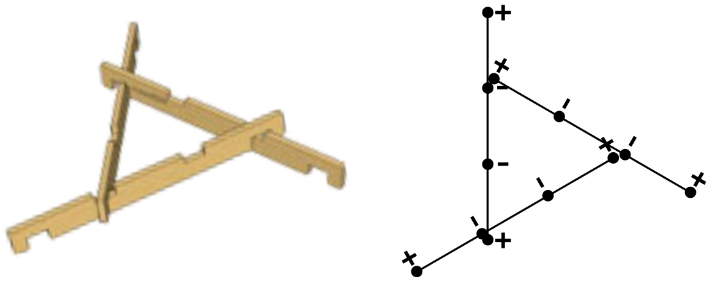


Picture n.9. A tessellation formed with a new pattern using the sticks with no curvature.

Leonardo’s tessellations.

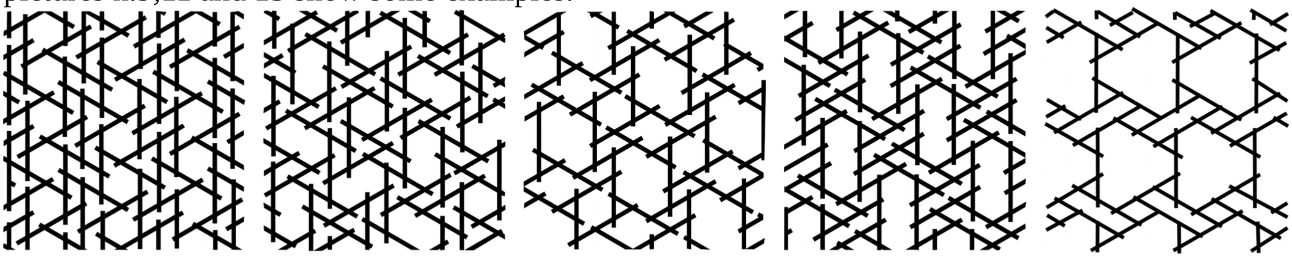
The patterns with which we can build the domes are deformations of tilings and tessellations of the plane. Next, we will analise with some of their characteristics without considering this deformations.

Formally, we could define a Leonardo Tessellations as a tiling of the plane formed by equal segments with four mutual contact points; the two furthermost with positive polarity, the two middle ones, dividing the sticks in thirds, with negative polarity, so that all contact points pair positive and negative together (picture n.10).



Picture n.10. Formalization of the sticks as segments with four contact point +--+.

The eleven patterns on picture n.7 are Leonardo tessellations, but they are hardly the only ones, pictures n.9,11 and 15 show some examples.



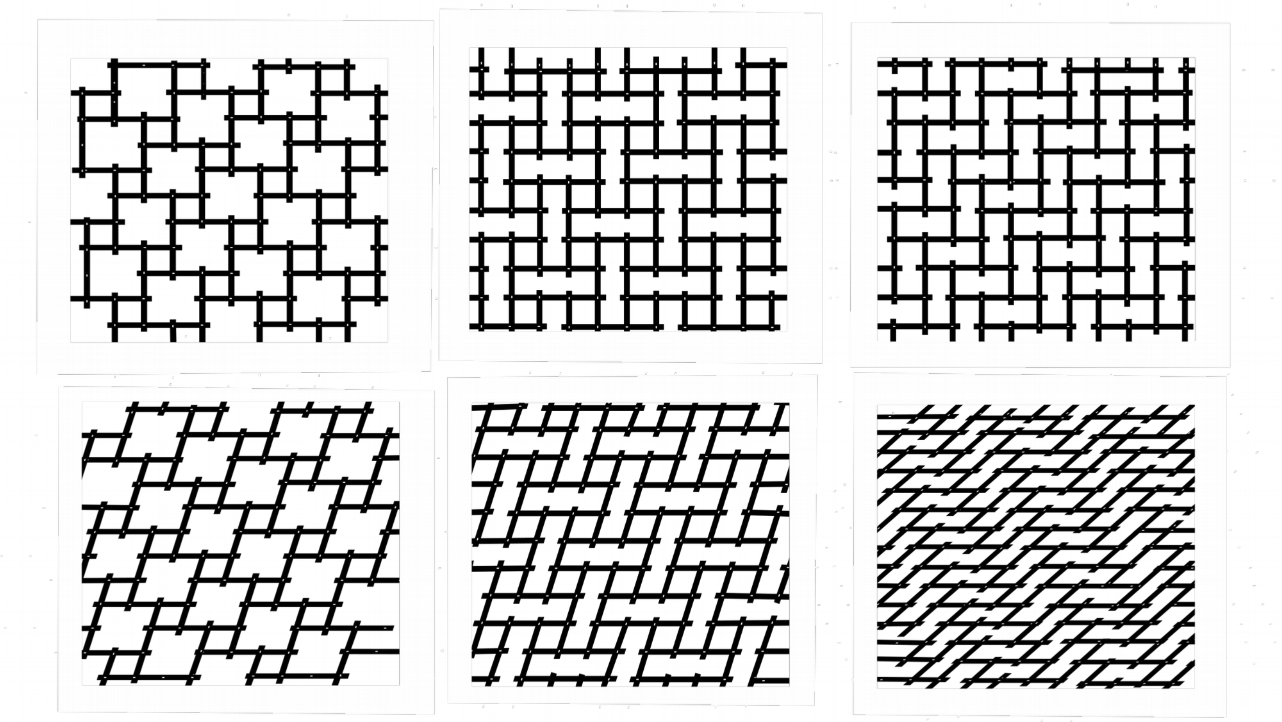
Picture n.11. Other patterns o Leonardo tessellations.

We can separate Leonardo tessellations in two: deformable and rigid.

Deformable Leonardo tessellations

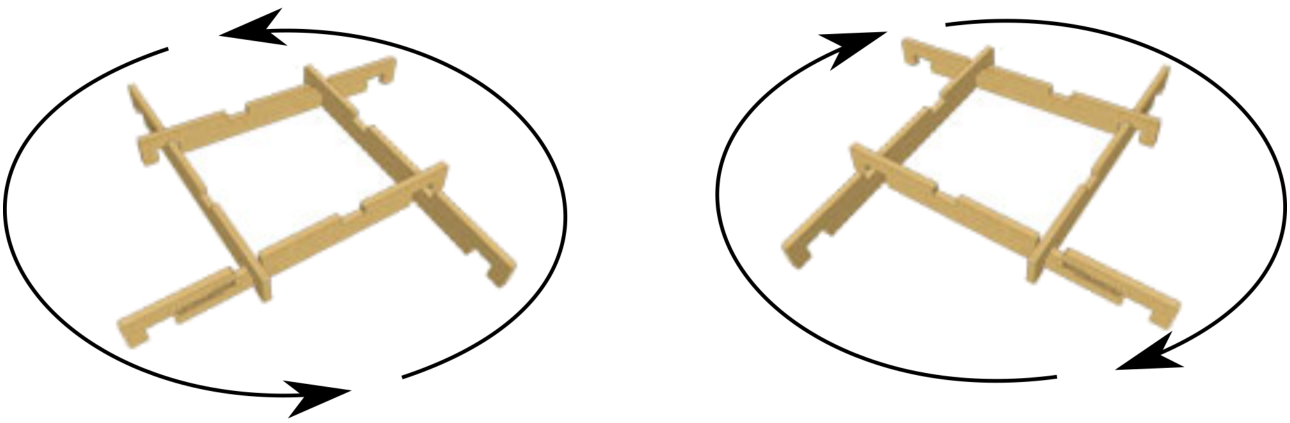
Tessellations 1, 2 and 3 in picture n.7 have the key characteristic of being formed entirely by parallelograms, their sticks are oriented in just two directions and the angle between these two directions is variable, as seen in picture n.12.

This is precisely one of the interesting activities in the building workshop. Once a completed dome is built and held up in the air, we can identify the two diagonals, then, diametrically opposed people can push and pull accordingly and witness the deformation of the whole structure. This deformation is not possible in other patterns, due to the existence of triangles and other specific polygons. This is a practical and easy way to understand this characteristic about parallelograms and its multiple applications.



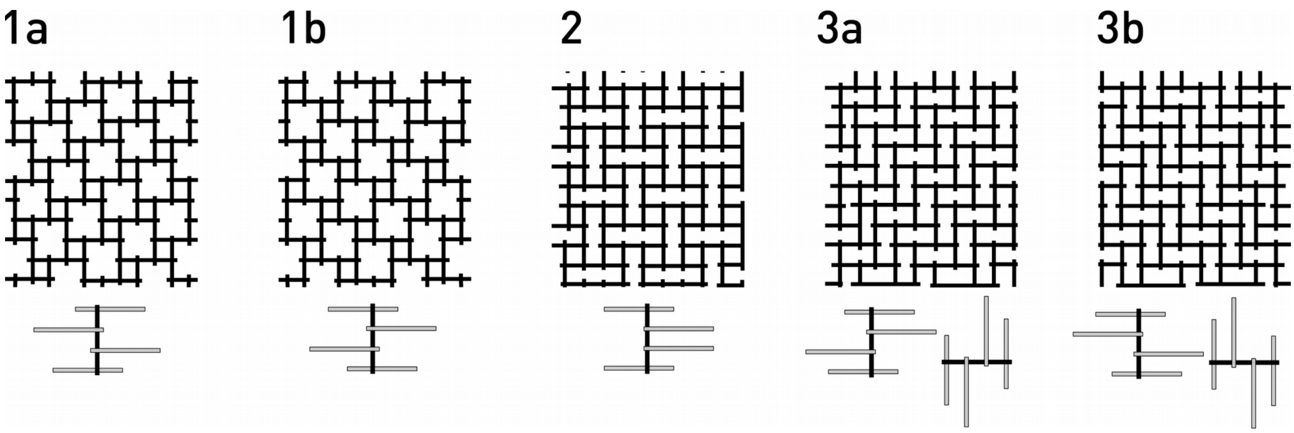
Picture n.12. Patterns and Leonardo tessellations n.1,2 and 3 showing angles between segments other than 90º.

At the beginning of building pattern n.1, we may realise that there are two ways of building a big square, which cannot be joined together. There exist, therefore, two symmetrical tessellations with bigger and smaller squares: 1ª and 1b (picture n.13).



Picture n.13. Two symmetrical ways of building the first pattern of the domes.

Tessellations n.2, however, is symmetrical in itself, while tessellation n.3 has two symmetrical versions again: 3ª and 3b. These five tessellations are the only ones with variable angles.



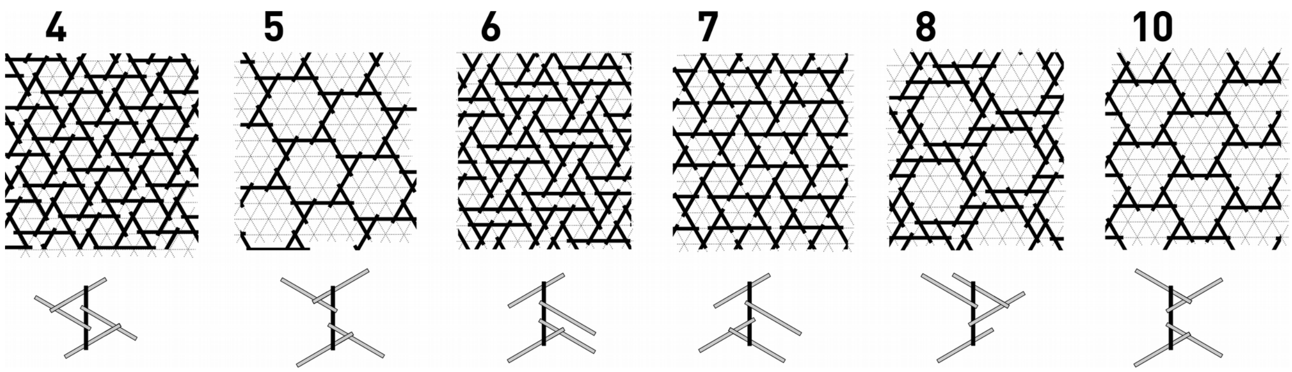
Picture n.14. The five deformable Leonardo tessellations and its corresponding configurations.

It is also worth focusing on how each four pieces are connected to one. These are the configurations shown on the bottom of picture n.14.

Fixed-angles Leonardo tessellations.

The first three patterns excluded, the rest of the tessellations which are possible with Leonardo sticks, have a particularity: all angles between pieces are either 60º or 120º. We may consider them and paint them in an isometric grid[4]

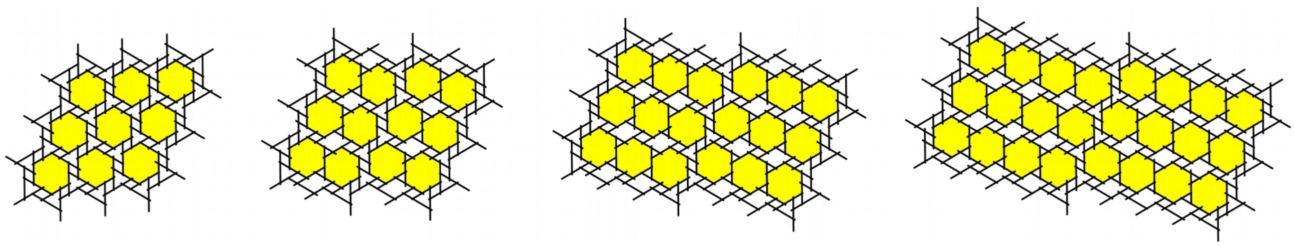
Particularly interesting are those patterns where all sticks have the same configuration. Those are the ones listed in picture n.15.



Picture n.15. Patterns with fixed angles and a unique contact configuration.

Except for pattern 10, neither of them have axial symmetry, and we must therefore also consider its mirror version.

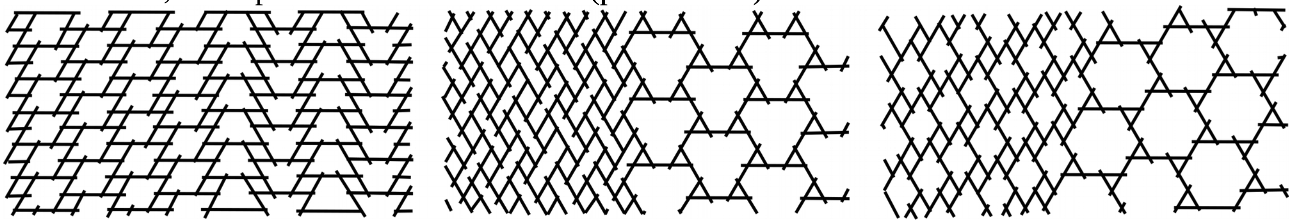
The number of Leonardo tessellations is infinite. This is a consequence of having found the infinite series shown in Picture n.16, built around pattern n.8. This pattern: hexagons surrounded by a series of rhombuses and triangles, can be generalized by surrounding one, two, three, four... hexagons with a series of rhombuses and triangles.



4 Also known as an axonometric grid, built out of equal equilateral triangles and used commonly in technical drawing

Picture n.16. The first four patterns of an infinite series of structures.

Furthermore, some patterns can be combined (picture n.17).



Picture n.17. Some patterns combined.

These mixed tessellations can only be translated in one direction. They cannot be considered regular because they don’t have a fundamental tile which can reproduce them with translations in two directions.

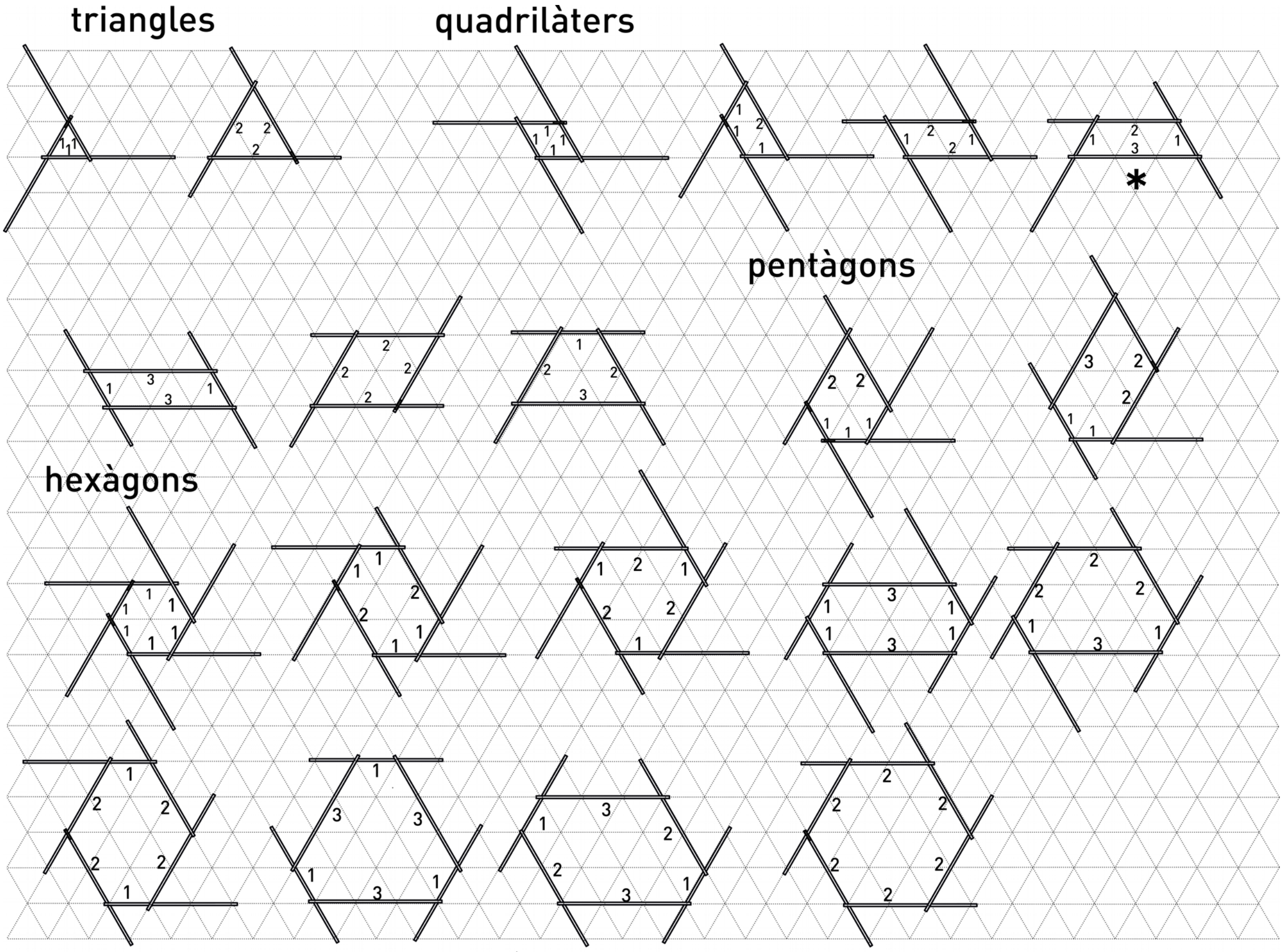
Leonardo’s convex polygons

A possible exercise is to build or draw all of the convex polygons which can be built with Leonardo sticks on an isometric grid. It may not be an easy task to find them all out, since there are twenty of them.

Looking for Leonardo tessellations which use these polygons, we have found new patterns, like the ones shown in pictures 9 and 11.

Working empirically, we have managed to build patterns using all of the twenty polygons on picture 18, except for the trapezium with side 1,2,1,3, marked with an asterisk (*) for which we have found no building tessellation.

One of our pending challenges is to formally demonstrate this and other questions referring to these tessellations.

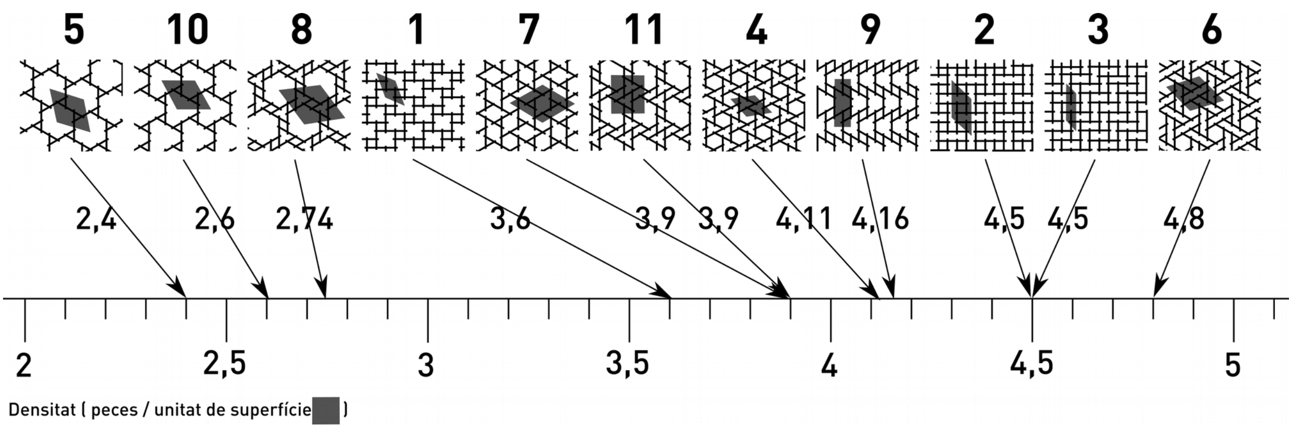


Picture n.18. The twenty polygons that can be built with Leonardo Sticks.

The density of Leonardo’s tessellations.

One of the questions that arise when comparing the eleven patterns is the different separation between sticks. In order to quantify that idea, we may define the concept of density as the amount of sticks in a given surface.

In order to calculate its density, we need to find the fundamental tiling behind each pattern, considering its minimal translations in two directions. By dividing the sum of sticks included in the fundamental tile and the surface that they cover (using the length of the stick squared as a unit) we may obtain the results in picture 19.



Picture 19. The eleven patterns used by the MMACA

Conclusions

LeonarDome is a multifaceted material in many aspects:

- Regarding its recipient: ages ranging from children of five to university students and families.
- Regarding the format: building workshop with 250 big pieces, geometry corners with 50 pieces and tabletop domes (with and without curvature).
-Regarding the transdisciplinarity of content and competences: the experience combines, not just geometry aspects and the teaching of values, it is also a great gateway into history, art and technology.

We must thank the MMACA educators, and everyone who has worked with this materials, for the enrichment that their ideas, new presentations and new activities have brought. We are sure we will continue to gather contributions, since the project's impressive welcome inside so many collectives.

The balance between the recreational and didactic aspects of LeonarDome, and the reflections and dialogues it provokes, confirm its strong competential character and indisputable pedagogical value.

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