## conference proceedings

international conference museums and dissemination centers math

Mathematics Awareness, Training, Resource & Information Exchange

2018 October 29-31

Cornellà de Llobregat - Barcelona

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#### Proceedings of the international III MATRIX CONFERENCE (ISBN: 978-84-09-12760-3)

Associació per Promoure i Crear un Museu de Matemàtiques a Catalunya (MMACA)

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# Proceedings of the international **III MATRIX CONFERENCE**

29th-31st October 2018, Cornellà del Llobregat (Barcelona)

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### Preface

During the 29th, 30th and 31st of October 2018, the 3rd MATRIX Conference (Mathematics Awareness, Training, Resource, and Information Exchange) took place at the Museu de Matemàtiques de Catalunya (MMACA) as the main venue, along with two additional venues: the Museu Agbar de les Aigües and the CosmoCaixa.

A total of five plenary lectures, sixteen conversation groups, a poster session and an exhibition with a sample of materials were organized during these three days, gathering together professionals working on math outreach, math museums, math education and math enthusiasts. More than 120 people from 22 countries took part and were engaged in the conference.

Since the 1st MATRIX Conference held in Dresden in 2014, and the 2nd edition in Leeds in 2016, the main objective of this meeting has been to share the experiences that are being carried out in the non-formal field of math education and generate new ideas for the role that museums of mathematics have in this field, as well as professionals of the math outreach and dissemination.

This cordial and constructive exchange has grown into a congress which had started as a simple working group that managed to expand and to inspire people from different parts of the world to join on board. We hope that the future editions of this project will continue increasing its size by encouraging and supporting the rise and bloom of new projects, new math promotion initiatives and new math museums that will surely emerge in countries throughout the world.



#### Acknowledgements

First of all, we would like to thank MoMath for being the promoter and pioneering organizer of MATRIX, a kind of meeting so necessary nowadays.

A successful MATRIX Conference requires the combined efforts of many agents, and we would like to thank all those who have helped us in the success of this third edition of the Matrix Conference. Support from thirteen sponsors and institutions, as well as public and private entities, has been decisive for developing and housing of the Conference. Support from universities and research centers through Scientific Committee was also valuable. Thanks must be extended to lecturers of inspiring and plenary conferences, and group conversations moderators, as they provide interesting content, methodologies and approaches. Moreover we are also proud and grateful for having a strong and reliable local community around MMACA with a remarkably positive attitude and constant support, what has been essential for the organization and the development of the conference.

Last but not least, we would like to thank all the attendees for being fully involved in the conference and sharing their experiences and points of view. They were the real protagonists. Thus, we hope to see you again at the 4th MATRIX Conference.

Organizing Committee Cornellà de Llobregat (Barcelona) July 2019

## Introduction

These proceedings provide a published report of the 3rd edition of MATRIX Conference, held at the Museu de Matemàtiques of Catalunya (MMACA), motivated by the success of the conference and the enthusiasm of participants. However, the aim was not a publication containing all what was said or experienced at the conference. That would have been a failure for sure: to tie three days of rich experiences in a finite number of pages would be an impossible task. Modestly, there was a double purpose: by one hand to keep track of the interest of plenary talks and topics of conversation groups; by the other hand, to give voice to several reflections and outreach experiences.

The first part presents the plenary conferences, with an abstract and a link to the videorecorded talk for each one. Some extended paper developing the content, by courtesy of the speaker, is also included.

The central part of this volume is devoted to contributions related with the conversation groups. A list of all conversation groups is included, with a short description of their scope. Then, following the order of topics in the list, descriptions of the conversation written by moderators or further reflections are included.

Finally, the third part includes papers describing outreach initiatives, namely real experiences, software or game resources and activities, presented by participants at the conference, in order to promote the interchange of good practices.

We would like to thank the authors for submitting their work, the group conversations moderators who contributed assuming the responsibility to lead discussions and participants who sent their contributions. We also thank the editors for leading and making possible the publication of this volume of proceedings. Received files were translated into suitable  $\&T_EX$  files by C. Luna resulting into this nice work, reflecting so many hours of dedication. M. Alsina wrote complementary reports to cover the range of topics and collect the spirit of conversation groups.

As a mathematical Scientific Committee from universities, we subscribe the enthusiasm to share discussions on Mathematics Awareness, Training, Resource and Information eXchange with professionals working on math museums and math outreach. Academic community has also a commitment with the society, therefore it has been a pleasure to be part of this project and we congratulate the Organizing Committee for the success of the conference.

Scientific Committee Cornellà de Llobregat (Barcelona) July 2019

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Part I

**Plenary Talks** 

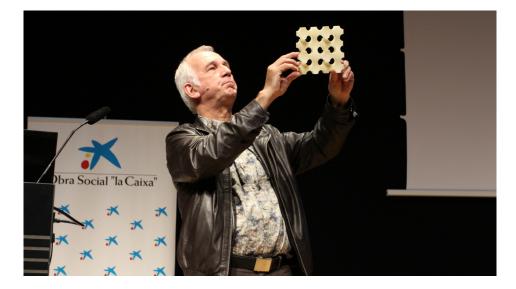
## **Art about mathematics**

By Rinus Roelofs (mathematician & artist)

**Abstract:** The main subject of my art is my fascination about mathematics. And to be more precise: my fascination about mathematical structures. Mathematical structures can be found all around us. We can see them everywhere in our daily live. The use of these structures as visual decoration is so common that we don't even see this as mathematics. But studying the properties of these structures and especially the relation between the different structures can bring up questions. Questions that can be the start of interesting artistic explorations.

Artistic explorations of this kind mostly leads to intriguing designs of sculptural objects, which are then made in all kind of materials, like paper, wood, metal, acrylic, etc. It all starts with amazement, trying to understand what you see. Solving those questions often leads to new ideas, new designs.

Since I use the computer as my main sketchbook these ideas come to reality first as a picture on the screen. From there I can decide what the next step towards physical realization has to be. A rendered picture, an animation or a 3D physical model made by the use of CNC-milling, laser cutting or rapid prototyping. Many techniques can be used nowadays, as well as many different materials. But it is all based on my fascination about mathematical structures.



Watch this talk: https://youtu.be/Sc4c\_70GUrM

## Catastrophes, diseases and crimes: risk prediction with mathematics

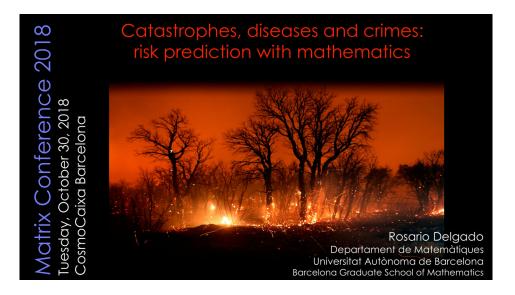
By by Rosario Delgado (UAB)

**Abstract:** Can we predict the risk of a catastrophe? Still more: can we assess how risk is affected if we take certain measures to try to avoid it? Which individuals are at greater risk of suffering from some illness? Can we assess the effect of risk factors or protection factors in our health? And the risk of criminal recidivism? Can we predict it from the available information of a person who goes out of prison and is released? What is the profile of the repeat offender?

Mathematics help us answer all these questions and many more! Bayesian Networks, introduced in the 20s of the last century as a graphic mathematical model that describes the probabilistic knowledge of the relationships between variables that affect a certain non-deterministic phenomenon, have shown to be very useful for risk assessment and profiling, in areas as diverse as economics, medicine, the environment sciences and ecology, the accidents with nuclear waste, or criminology, among many others.

This talk introduces the tool of Machine Learning, through examples, both academic and real, for a public not specialized but interested in science in general.

Watch this talk: https://youtu.be/uOHra4sMNHI



#### **Extended Abstract**

Can we predict the risk of a catastrophe? Still more: can we assess how risk is affected if we take certain measures to try to avoid it? Which individuals are at greater risk of suffering from some illness? Can we assess the effect of risk factors or protection factors in our health? And the risk of criminal recidivism? Can we predict it from the available information of a person who goes out of prison and is released? What is the profile of the recidivist offender?

But what do we mean by the word "risk"?

In general, **risk** is any event that can have a negative impact. In our context, however, the word refers to the **probability** of suffering the consequences of any event that can have a negative impact. **Assessing the risk** in a given situation consists in estimating or predicting this probability. How is this estimate made? The traditional way to do this is the following, which will be explained by an example.

**Example:** Armageddon is a biblical term used to refer to the end of the world through catastrophes. This is the title of the 1998 American science fiction disaster film directed by Michael Bay and starred by Bruce Willis, Ben Affleck and Liv Tyler, whose script tells us that NASA discovers that a giant meteorite approaches the Earth, and that its trajectory will make it collide with our planet. As the only idea to try to avoid catastrophe, they send a group of blue-collar deep-core drilling experts to the meteorit in order to make it explode with a bomb, so that it does not impact the planet. The world was confronting a truly massive risk, a truly CATASTROPHE!!

The traditional Approach (Impact-Based Risk Measure) to estimate risk is to compute it by multiplying the probability of the event with negative consequences for a measure of its (negative) impact.

This type of measure can be useful to prioritize risks (the bigger, the more serious the risk is), but in practice it is difficult to use. Why? Because we can not calculate the probability that the event will take place nor the measure of its impact, without making an analysis of the variables that may have a relationship. For example, if, according to NASA scientists, meteorite is in direct trajectory with the Earth, this would say that the impact is sure to occur and, therefore, that the probability of the event occurring what causes the risk is 1. We all will agree on this. But if so, what is the point of sending the drillers in a spaceship, risking their lives to try to place a bomb on the meteorite?

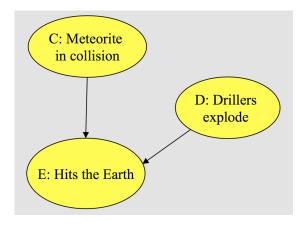
Are we in a dead end? Fortunately for the inhabitants of our planet, no! And it is that the probability that the meteorite impacts the Earth is conditioned by other events, such as human intervention to try to explode the meteorite before the impact. Therefore, it is pointless to consider the probability of the impact taking place without considering these other events that may affect it. This simple fact makes us see that the classic way of calculating the risk (Impact-Based Risk Measure) does not help us in this case. How can we, therefore, estimate the real risk of this catastrophe on our planet? Mathematics help us do it. Bayesian networks, introduced in the 20s of the last century as a graphic mathematical model that describes the probabilistic knowledge of the relationships between variables that affect a certain non-deterministic phenomenon, have proved its great utility for risk assessment and profiling, in areas as diverse as economy, medicine, environment and ecology, accidents with nuclear waste or criminology, among many others. Bayesian networks consist of a directed acyclic graph (DAG) and some parameters, which are the probabilities of the probability distribution of the variables (conditioned to the parents, which are the variables sending an arrow to it). In our example, how can they help us estimate the risk caused by the meteorite? Building a mathematical model that will allow us to make the estimate. To build the mathematical model, in the first place, a graphical representation of the relevant variables in our situation, and of the relations of dependence between them, is made. The variables we have are:

**C: Meteorite in collision trajectory.** Is the current trajectory of the meteorite of collision with the planet? False/True

E: Meteorite hits the Earth. Will the meteorite really impact the Earth? False/True

**D: Drillers explode the meteorite.** Are the drillers able to explode the meteorite? False/True

And the relationship between them is represented in Figure 1, where the arrows indicate the dependency relationships.

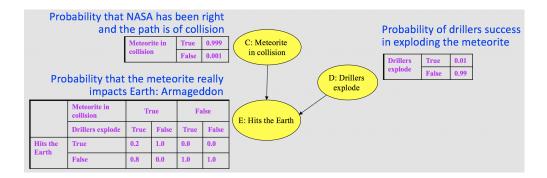


Secondly, we need to know the parameters of this mathematical model, which are the following probabilities (in general, these probabilities are learned from a database, but they can also be given by the experts, as in this case):

1. Probability that NASA has been right and the current meteorite path is really of collision with the planet: 0.999 (big but not one, since there is always the possibility of error in calculations). It is the probability that collects all the information we have about the variable C.

- 2. **Probability that the drillers are successful in exploding the meteorite:** 0.01 (it is small because the task is really very complex and is carried out to the desperate). It is the probability that collects all the information we have about the variable D.
- 3. **Probability that the meteorite really impacts Earth:** here we have to take into account the other two variables, which affect the variable E. That is, the information we have of variable E (which is finally the one that interests us) is conditioned by the other two variables:
  - 3.1 **If C = False:** 0 (if the trajectory is not really of collision, nothing happens, regardless of what the drill team does).
  - 3.2 **If C = True, D = False:** 1 (if the trajectory really is of collision and the drill team is not successful, the catastrophe can not be avoided).
  - 3.3 **If C = True, D = True:** 0.2 (if the meteorite's current trajectory is of collision, even if the drill team gets the meteorite to explode, it might be that a large piece of it hits the Earth, causing the catastrophe).

This information is summarized in the probability tables (for variables C and D) or conditional probability tables (for variable E), in Figure 2.



We could make the model more complex, but this is enough to serve as an example. Now that we have built the model, which is a Bayesian network, we can use it to estimate the risk of catastrophe in different scenarios.

**Scenario 0.** We calculate the "a priori" probability of catastrophe, that is, we estimate the risk that the collision will actually occur with the extra information we have at the moment, which is none. For that, we use the **Law of Total Probability**.

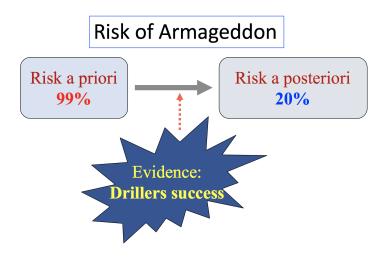
$$\begin{split} P(\mathsf{E} = \mathsf{True}) &= P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{True}, \, \mathsf{D} = \mathsf{True}) \, P(\mathsf{C} = \mathsf{True}, \, \mathsf{D} = \mathsf{True}) \\ &+ P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{True}, \, \mathsf{D} = \mathsf{False}) \, P(\mathsf{C} = \mathsf{True}, \, \mathsf{D} = \mathsf{False}) \\ &+ P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{False}, \, \mathsf{D} = \mathsf{True}) \, P(\mathsf{C} = \mathsf{False}, \, \mathsf{D} = \mathsf{True}) \\ &+ P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{False}, \, \mathsf{D} = \mathsf{False}) \, P(\mathsf{C} = \mathsf{False}, \, \mathsf{D} = \mathsf{False}) \\ &= 0.2 \times 0.999 \times 0.01 + 1.0 \times 0.999 \times 0.99 + 0.0 \times 0.001 \times 0.01 + 0.0 \times 0.001 \times 0.99 \\ &= \mathsf{0.991008} \quad \Leftarrow \mathsf{Risk a priori} \end{split}$$

Note that probabilities of the type P(C = True, D = True) have been calculated as product of P(C = True) P(D = True), since the variables C and D are independent to our model (without entering into the details of this fact, intuitively we can say that knowing the value that takes one of the variables does not affect the probability with which the other variable takes its possible values; this is an assumption of our model, but it seems quite reasonable taking into account the meaning of the variables).

**Scenario 1.** We have a new information, an evidence: we have just learned that the drill team has been successful to explode the meteorite. Now we can re-estimate the risk, taking into account this information, that is, we can calculate the a posteriori probability, using the **Law of Total Probability Conditional**, of E = True, conditioned to the evidence D = True:

$$\begin{split} P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{D} = \mathsf{True}) &= P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{True}, \, \mathsf{D} = \mathsf{True}) \, P(\mathsf{C} = \mathsf{True} \, / \, \mathsf{D} = \mathsf{True}) \\ &+ P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{False}, \, \mathsf{D} = \mathsf{True}) \, P(\mathsf{C} = \mathsf{False} \, / \, \mathsf{D} = \mathsf{True}) \\ &= P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{True}, \, \mathsf{D} = \mathsf{True}) \, P(\mathsf{C} = \mathsf{True}) \\ &+ P(\mathsf{E} = \mathsf{True} \, / \, \mathsf{C} = \mathsf{False}, \, \mathsf{D} = \mathsf{True}) \, P(\mathsf{C} = \mathsf{False}) \\ &= 0.2 \times 0.999 + 0.0 \times 0.001 = \mathbf{0.1998} \quad \Leftarrow \mathbf{Risk a posteriori} \end{split}$$

We see that the risk has dropped sharply, which means that the evidence we have goes against the catastrophe. The result is also logical now: if the drills succeed in completing their mission, the probability of catastrophe is that there is no error in the calculations of NASA scientists, which is 0.999, and that despite the success, a large part of the meteorite still impacted the Earth, which is 0.2. And if that happened, we could not do anything else. Fortunately, this will only happen with probability 0.1998, less than a 20%!



These models can be also used to answer questions related with

#### • Healthcare:

What are the variables that increase the risk of suffering a disease/accident (risk factors)? Which ones reduce the risk (protection factors)?

Which is the risk of an individual of suffering from a disease/accident based on his/her features (diagnosis)? To what extent the diagnosis is sensitive to certain features? (sensibility analysis)

Which are the profiles of individuals at greater risk of suffering from the disease/accident?

#### • Criminology:

What are the variables that increase the risk of be victim/author of a crime (risk factors)? Which ones reduce the risk (protection factors)?

Which is the risk of an individual of being victim/author of a crime based on his/her features?

Which are the profiles of individuals at greater risk of being victim/author of a crime?

Answers to these questions allow us to influence the prevention of the disease/accident or of the crime, to improve the diagnosis and profile, and to optimize resources management.

By way of conclusion, we will say that Bayesian networks are probabilistic mathematical models of Machine Learning that can be used for risk assessment and for profiling. These models are learned from a database set, although they also can be learned from experts, from which they are also validated, obtaining their predictive accuracy. Bayesian networks are applicable in many fields: criminology, medicine, disaster prevention, climate change, occupational hazards, traffic accidents, ...

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## The Big Data phenomenon

By Lali Soler (Data Analytics Business Manager)

**Abstract:** In this talk we see how the convergence of three relevant trends (big data technologies, artificial intelligence and interactive interfaces) are opening a great amount opportunities in the new data economy, and are changing the way organisations act. Also, some time is devoted to talk about the maths within the Big Data phenomenon as well as to point out some possibilities to bring them in an interactive space.

Watch this talk: https://youtu.be/UJgudkmJDtl



#### The Big Data Phenomenon



### Impact Evaluation in science vocation

By Sergio Marco (Everis), Digna Couso (Crecim – UAB) and Lluís Noguera (CosmoCaixa)

**Abstract:** FECYT coordinates this panel that will present different Spanish initiatives to evaluate the impacts of non-formal science education and science outreach activities. Firstly, Gonzalo Remiro from FECYT sums up their experience in evaluation of these kind of activities, with special attention to their publication "Toolkit for evaluation of scientific culture activities". Following, Digna Couso from CRECIM-UAB, an university professor expert in science education, will show the results of their investigation in the evaluation of non-formal science education activities at qualitative level, including the evaluation of experimental workshops for secondary school students (project with PCB-UB) and the evaluation of school-research collaborations (project with Fundación Elhuyar). Finally, Sergio Marco from Everis will present de study "How to stimulate a scientific mind?" developed by FECYT, Obra Social "la Caixa" and Everis that defined and implemented an impact evaluation system that represent the extent to which the interest in studying STEM increases in the students taking part in outreach activities and identify the key influencing factors in young people's career choices.



https://youtu.be/OGrVOXUL26E

Watch this talk:



## Role-play of coordinates and function dance

By Grup Cúbic and MMACA

**Abstract:** There are different types of resources to teach mathematics, in this workshop we propose a very special one where the students themselves and teachers become resources taking on the role of mathematical beings and behaving coherently with this role in the framework of small role-plays. In this conference we will be points in a coordinate system. To finish we have prepared a small surprise... we're waiting for you!

Watch this talk: https://youtu.be/IJ-HYivDoTg



Part II

**Conversation Groups** 

## **Conversation Group Abstracts**

By MMACA

Matrix conference promoted discussion forums, organized as conversation groups, that proved to be very productive and interesting, but difficult to survey as written contribution. Next a list of all the topics and a brief description is included.

#### **CG1: Museum and School**

«We don't teach, but they learn»

**Moderator**: Albrecht Beutelspacher

The learning models in a museum (exhibitions and workshops) are not the same than in school. Dynamics, time, relationship and communication are different.

At museums, we don't teach. We make self-experience possible.

#### CG2: Mathematics and children (0-8)

«There are not small mathematics, there are not small mathematicians»

#### Moderator: Montserrat Torra Bitlloch

Mathematics experiences have to be intense, challenging, stimulating for teachers and students at all the educational steps. How can we support this goal from the museum?

#### CG3: Hands-on and virtual hands-on

«Is virtual virtuous?»

#### Moderator: Ana Cristina Oliveira

The presence of technology in all social areas (school, home, relationships, information...) is continually increasing and gaining new competences. The math component is enormous and not always evident. Which and how much technology is needed in the museum? What is the purpose?

#### CG4: International Math Week

«Let's do something together»

#### Moderator: Eoin Gill

The popularization of mathematics through exhibitions, fairs, workshops or individual events is a widespread practice in many countries and that sees all of us involved. Is it time to organize collective actions, exchanging experiences and activities? Which models would allow us to break the barrier of silence?

**Conversation Groups** 

#### CG5: Museum and Teachers training

«Don't be so formal!»

Moderator: Montserrat Alsina i Aubach

The initial and ongoing training of teachers is conducted in different ways in different countries, but it is difficult to see the entities that educate outside the formal circuit, from kindergarten to university, involved, even when the goal is skills based learning.

Do we have alternative models to offer?

#### CG6: Museums and scientific vocation

Moderator: Francesc Rambla i Marigot

Projects to stimulate STEM vocations are taking place in many countries, to compensate for the lack of professional figures and the demand for an increasingly technological industry. Museums are called to be interlocutors of these projects. What are our peculiarities? How can we measure (and claim) our impact and contribution?

#### CG7: Statistics

«In the museum and for the museum»

«Mummy, I want to be a scientist!»

Moderator: Pepus Daunis-i-Estadella

The statistic (and probability) is a matter of mathematics that is more present in the social reality than within museums. Is it possible to develop meaningful modules that will allow us to increase the skills to better understand information and take better decisions?

On the other side, do the museums use the property and efficiency of the instruments and the statistics predispose to evaluate the effectiveness and dimensions of their own social and educative skill?

#### CG8: Math, Museum and People Dissemination

«Go wider, go deeper!»

#### Moderator: Jordi Deulofeu

In recent years, mathematical dissemination shows a greater social presence. Books, articles and interviews in newspapers and magazines, collections of games, some movies, fleeting but repeated television appearances ... are more frequent. Can we take advantage of it to be able to impose a more realistic view of mathematics and mathematicians, beyond the easy stereotypes that persecute us?

#### CG9: In-E-motion

«Movement and emotion stimulate creActivity»

#### Moderator: Maurici Carbó

Simulations, applets, transforming shapes, puzzles, fractals, paths, strings... Dynamic images generate emotions and stimulate personal investigation.

#### CG10: Museum and Reality modelling

*«Banner at the exit of the museum: "You are now entering the real museum of mathematics"* 

#### Moderator: Pelegri Viader

From a certain point of view, if we succeed that our educational discourse is focused on elements of daily life we can reach a greater, less expert, but no less curious audience. From another point of view, reality is too complex to be contained in a single module. Modelling without losing the essential elements that we want to mathematise is the big bet.

#### **CG11: Research in Statistics**

«Is the Big Data a Gentle Giant?»

#### Moderator: Marina Brassó

BigData is breaking into world of statistics as a real challenge, but how can we convert this huge landscape into a museum hands-on module? We are able to use that technology in our benefit?

#### CG12: Support for new www.seums! Well, it's not so hard! I mean: it's hard, but is fun!»

#### Moderator: Fernando Blasco

Is it really so difficult and expensive to build a new mathematics museum? Which elements are essential? How to stimulate demand in the area?

#### CG13: Sharing of exhibits ideas

«Sharing is caring»

#### **Moderator**: Daniel Ramos

In Dresden's first Matrix conference, Imaginary proposed a collaboration protocol between mathematical museums. Although not formalized, a tacit agreement was in fact accepted and contributed to the realization of excellent collaborative experiences. With this baggage and more available connection channels, how can we widen and generalize such collaborations?

#### CG14: Emotion vs Fun: a transforming museum

«I had a lot of fun. I have a lot of questions»

#### Moderator: Cindy Lawrence

The emotional (hearts-on) and playful aspects play an essential role in determining the success of an exhibition, but hide the risk of trivializing content and objectives. We must promote a seductive science, which changes the commonly accepted view of mathematics and maintains this new interest for a long time.

Conversation Groups

#### CG15: Museums, Big or Great?

«Size doesn't matter»

#### Moderator: Tim Nissen

Does it make sense to discuss the ideal dimensions of a mathematics museum? Are we able to decide or suggest the best model or different models are equally possible and effective? Which elements are essential for a good activity?

#### CG16: Museum and University

«How could I explain it to you?»

#### Moderator: Pere Pascual Gainza

Popularization of theory advances in mathematics is an important social goal. Is the museum the right place to do it?



## **CG1: Museum and School**

By M. Alsina (editor)

«We don't teach, but they learn»

The learning models in a museum, mainly based on exhibitions, workshops or other events, are not the same than in school. Their development, dynamics, time, relationship and communication are different. Thus, focussing on museums, the discussion on this group took as beginning point the statement: at Museums we don't teach, we make self-experience possible.

Participants contributed from their own experiences, guided by Albrecht Beutelspacher as moderator.

Tiago Hirth, one of the founders of the Circo Matemático, a Maths Outreach project in Portugal, shared his experience at https://circomatematico.wordpress.com and remarked interesting points of their work and how they approach the general and scholar public. He also worked closely with the National Museum of Natural History and Science and their maths exhibits, among other science and maths outreach and popularization projects he was talking about.

Noel Jackson focused on the importance of making problems hands on. Museums play a role: how classic maths problems can be made engaging for teenage audiences by increasing the scale and making the students become part of the problem.

Erika Berenice Roldan Roa shared how allowing students to select a project to work on, after giving them the fundamentals of coding and logic, had been for her the most exciting learning experience leading a workshop.

Vinay Kathotia at Manchester Metropolitan University explained a joint work with Ricardo Nemirovsky, consisting of developing open-ended informal learning experiences that foster and support mathematical expressiveness and agency. These interweave creative activity, technology, and pervasive mathematical ideas such as curvature and symmetry. See their extended contribution in this volume.

Guido Ramellini, from MMACA, deeply believes that formal and non-formal learning approach must collaborate. School mechanisms must not be repeated in a museum or a Science Centre. We should not teach, but offer open learning situations. Often, a visit to an exhibition provokes a positive emotion (hearts-on) that can be the seed of a new interest on the STEM. See his extended contribution in this volume. Quoting Galileo Galilei: You can teach nothing to a human being; you can only help people to *find the answer inside them*.

Also Jorge Silva, Yordan Hodzhev and Carlos Seara were interested in the discussion.

## **Critical Experiences of Expressive Mathematics**

By Vinay Kathotia and Ricardo Nemirovsky (Manchester Metropolitan University)

#### Introduction

Striving to foster and support mathematical expressiveness and creativity, we are developing and studying open-ended learning experiences at Manchester Metropolitan University. These interweave open-ended activities, use of technology, pervasive mathematical ideas (e.g. curvature, dimension, symmetry), and tangible processes that embody them (e.g. sewing, slicing, folding).

A motivating principle for this work is *the affective turn*, whose core tenet is that cognition/reason and feeling/emotion cannot be separated [1, 6]. The mathematical learning experiences are intended to be playful, thoughtful, collaborative, and rather than being driven by formal assessment, involve a sharing and celebration of insight.

But there are significant tensions at play here, for example, the contested nature of what constitutes mathematics and human understanding of mathematics [5], the tension between activities being open-ended and goal-oriented, between emergent and teleological learning [4], the divide between informal and formal learning and whether the proposed activities could be adequately or legitimately integrated in curricular learning, and even when they sit alongside traditional learning activities, whether such experiences can displace existing negative perceptions of mathematics. A key aspect of our work is to study these tensions, attend to the role of various constituent elements in these tensions, and explore their affordances and constraints in accommodating or resolving these tensions. Thus the word "critical" in critical experiences serves two purposes, that such experiences could be crucial for mathematical development and that we intend to look at them with a critical eye.

#### Design

Expressive mathematical experiences have been conducted in many settings. Below are two examples with quite different formats.

*Geometry and the Imagination* [3], variously run as a 2-week summer workshop in Minneapolis in 1991, or as a semester-long course in Berkeley in 1995 (led by Bill Thurston), and earlier

in Princeton (led by Peter Doyle, John Conway, Jane Gilman and Bill Thurston), involving a mix of students – high school students, college undergraduates, high school teachers and college teachers, culminating in an end-of-course *Geometry Fair*.

MoMath *Symmetry Code*, a 3-hour activity for students in grades 7 through 12, delving into programming, symmetry, and leading to the creation of their very own MoMath logo. https://momath.org/upcoming-events/symmetry-code/

There are a number of overlapping variables at play – the duration of the experience, the age range of those involved, mathematical or motor skill prerequisites, the role of technology, how structured/unstructured or goal-driven/open-ended the activities are. Our aim is to trial and study a variety of experiences, clarify the affordances and constraints of key variables, develop a framework for the related mathematical and pedagogical ideas, and share examples of practice.

#### **Motivating principles**

#### 1. The affective turn

While philosophers have separated since ancient times the realms of reason and emotion, over the last 15 years, throughout the social sciences, theorists of *the affective turn* [1, 6] have advanced the theses that: (a) these two realms cannot be separated: ideas devoid of affects, or emotions without cognitions, are theoretical constructions nowhere to be found in actual life and (b) these two realms are not individual phenomena: they are inter-subjective, circulating across social events, technologies, and living bodies [2].

From these perspectives, mathematics is a very important case because no other discipline has historically undergone a more radical separation between intellect and emotion, to the point that mathematics has often been presented as reason itself, contributing to the popular perception of mathematics as "cold" and "unfeeling". This despite the evidence that for numerous mathematicians, scientists, and artists (G.H. Hardy, R.P. Feynman, M.C. Escher to name a few) the mathematical experience is a profoundly aesthetic experience, intertwining simplicity, beauty and joy.

One goal of our approach is to break the artificial and damaging divide in mathematics between the realms of cognition and emotion. Rather than engaging with mathematics split in terms of intellectual failure and emotional anxiety, the main challenge in our field is to provide experiences and learning environments amenable to the creation of new images of mathematics – images that are inclusive and inspiring – where participants encounter mathematics as a lively and open synthesis of experiences, positively unifying cognition and affects, and in which *both* matter.

#### 2. Agency and expressiveness

A second and intertwined goal of such experiences is to support participants in seeing and using mathematics as a fluid and playful medium that is susceptible to personal and collective expression, not unlike the work in arts and crafts disciplines. Tools and technologies have always been instrumental for the creative and improvisational agency of artists and craftspeople, and they are, likewise, necessary for mathematical tinkering and design. An associated tension is that becoming fluent with many tools and technologies involves long-term training processes and regular skillful practice. One hypothesis that we are beginning to explore is about the use of new generations of computer-based technologies may allow for creative expressiveness with materials and tools even without a huge investment in time and effort seeking to develop their mastery. Developing new understandings of the interplay between the mathematics, material and tangible processes, and technology is a major goal of the research that we are pursuing. To this end, rather than focusing on formal assessments, we strive to document episodes involving curiosity, personal insights, memories, challenges, collaborative practices, reflecting, and sharing.

#### 3. Critical tensions

As mentioned in the introduction, there are significant tensions in the design and realisation of such experiences and a key aspect of the proposed work is to attend to, study and respond to these tensions. To underscore this aspect we briefly discuss a couple of overlapping tensions.

How open-ended could and should such experiences be if we are, on the one hand, attempting to develop the use of particular mathematics, materials and technology (say the interplay of curvature, basket-weaving and laser-cut patterns as in the "Basket Weaving and Curvature" programme that was part of the InforMath project [4], or the use of particular software or design principles in MoMath's *Symmetry Code*), while, on the other hand, also giving learners freedom of exploration so as to support their agency and expressiveness? How would this play out or be feasible if such experiences were to be made more mainstream and 'delivered' in classrooms, subject to strenuous demands of curricular mandates and compulsory testing? Or would such experiences have to be outside of the mainstream, almost being defined as "informal" or "enrichment" by their complementarity?

Are such experiences able to counter prior negative images and self-images learners may have developed about mathematics and about themselves as mathematicians? Do such experiences create different co-existing notions of mathematics, or if they do modify existing images, how lasting and empowering are these? Intertwined with the above is the fundamental and contested terrain of what constitutes mathematics. Not just the traditional divides of pure versus applied, cognitive versus affective, but also consideration of the ethical, social and economic values and purposes of mathematics. These are longstanding and live tensions, and exploring how they manifest themselves in programmes of expressive mathematics is a critical element of the proposed work.

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# What is "non-formal" education?

By Guido Ramellini (MMACA)

#### What is "non-formal" education?

The answer, from the booklet of Erasmus+ Project *MathSpaces*, is:

«Any organized educational activity outside the established formal system, whether operating separately or as an important feature of some broader activity, that is intended to serve identifiable learning clienteles and learning objectives.»

#### Differences between the formal and non-formal approach

- **Timing:** rigidity to previous predictions. School activities are performed in an allotted time, so flexibility is minimal. In the N-F.A., we can try to get the maximum of flexibility and each participant can stress the time necessary to perform a task in relation with interest, acknowledgment or ability.
- **Contents:** fidelity to the curriculum. The N-F. A. is often based on disciplinary contents that are not included in the school curriculum. Some people working in the S.Cs consider that their **entire** proposal must be extracurricular, so that a visit to a museum could be an original cultural experience, absolutely different from a school activity.
- **Dynamics:** individual/collective tasks, results, verifications... In schools, the average student progress often determines the pace of the class, condemning those who are not equipped to be left behind and those who have more will and/or capacity to be frustrated. In S.Cs' exhibitions, each participant, even though during the task a cooperating group has been formed, decides when to stop. A characteristic of nonformal education is the heterogeneity of the groups that are formed, sometimes around a single exhibit. The result is a more "real", less predictable communication.
- **Relationships:** transmission (from one who knows to one who doesn't know) and collaboration (community of learners). In a N-F.A. situation, the roles are not previously assigned; the exchange of information is free and bi-directional; the authority is earned on the field and is still temporary.
- **Communication:** quantity and type of information necessary to performing the task. In school, student's performance is supported by a big amount of information: professor, text-book and supplementary means. In a S.C. exhibition starting information is often reduced to the simple rules needed for developing a task. Facilitators should help as little as possible; to suggest, never to solve; to accompany, never to guide.

Deepening or enrichment of notions are delegated to another situation, as a workshop (in the museum or at school), individual research or, why not?, a later school-based intervention, a bridge between formal and non-formal education.

### Common elements of formal and non-formal approach

This is the point: **we deeply believe that formal and non-formal learning approach must collaborate**. In a Science Centrum we must not repeat school mechanisms. We should not teach, but offer open learning situations. Often, a visit to an exhibition provokes a positive emotion (hearts-on) that can be the seed of a new interest on the STEAM.

In schools, we must take advantage of every opportunity that allows students to develop on their own interests and abilities, alone or in a group, and encourage them to make their investigation available to the class (minds-on).

Furthermore, there are important aspects that must be developed both in school and science centrums.

A significant and long-lasting learning cannot be separated by a powerful desire to learn on behalf of learners. This means that we have to offer activities that stimulate interest and autonomy, that encourage them to look for and to communicate the results of their work.

We can propose open problems, in which more than a solution is possible and strategy can be discussed. A good example is this Canadian open-source page, (*Which one doesn't belong?* - http://wodb.ca/index.html).

Open challenges allow that sometimes users, free to spend as much time with it as they want, can find an original solution or a strategy we have not foreseen. That brings a deep personal satisfaction that we have felt ourselves, sometimes with materials we have known for years.

The goal of promoting personal/group investigation can be pursued through computer supported activities too.

In the same line, it could be interesting to show some of the historical solved (recurring fractions, sum of infinite sequence of square or triangular numbers...) and/or unresolved problems (for ex., Fermat's conjecture, the squaring of the circle, the trisection of the angle...), accompanied by physical or virtual hands-on materials that highlights the simplicity of the utterance and the difficulty of the solution.

The principle goal of education in the schools should be creating people who are capable of doing new things, not simply repeating what other generations have done. – Jean Piaget

We should definitely opt for a competency-based learning; the mere transmission of content and skills is not enough.

«Competency-based learning is learner-focused and works naturally with independent study and with the instructor in the role of facilitator. This learning method allows a student to learn those individual skills they find challenging at their own pace, practising and refining as much as they like. Then, they can move rapidly through other skills to which they are more adept.»- https://en.wikipedia.org/wiki/Competency-based\_learning

# CG2: Mathematics and children (0-8)

By M. Alsina (editor)

«There are not small mathematics, there are not small mathematicians»

The conversation group dealt with mathematics and children, up to eight years old. At this early age, mathematics experiences have to be intense, challenging, stimulating for teachers and students at all the educational steps. The aim of the conversation group was to discuss the question: how can we support this goal from museums?

The moderator, Montserrat Torra, highlighted that thinking that children only start learning mathematics from the age of 8 is wrong. Up to 8 years, children learn important mathematics too. So, very valuable mathematics experiences should be promoted from early age. The big challenge is to interpret what they do and what they say. See also next paper by M. Torra in this volume of proceedings.

Some participants based their contributions on examples of activities they already proposed to children.

Thus, Jehad Abualkbash presented *Museum (3, 4 & 5)* as the first of its kind in the world covering the curriculum proposed for third, fourth and fifth grades textbooks and focusing on concepts of mathematical education. It gives to the students in those levels full opportunity to enjoy going through the exhibits, in a period that does not exceed three hours, in an atmosphere that stimulates their curiosity and learning.

In a complementary direction, Yordan Hodzhev presented *Open Geoboard*. It is a wooden geometric board and a platform, which children use to explore and learn mathematical concepts like figures, forms, symmetry, 2 and 3 dimensional space and many more. It can be used as a learning material at home and in the classroom by kids and their parents, teachers and educators. The geometric board and different rubber bands, wooden pegs, templates and other elements are the tools for making diverse forms, figures, objects, art and many more in two and three dimensional space. Changing the template on the board makes it a basis for board games like Ludo, Chess, Maze etc.

Other participants, as Jorge Silva, Emilia Vasilescu, Elena Yakubovskaya and Carlos Seara, were also interested in this topic.

# When do the children begin to learn mathematics?

By Montserrat Torra Bitlloch (Departament d'Ensenyament, Generalitat de Catalunya)

## When do the children begin to learn mathematics?

Knowledge about early childhood has improved in recent times. Neuroscience [1] tells us that children learn since they are born.

And they learn mathematics:

- When moving and finding reference points, they learn to orientate themselves to space.
- When they are evaluating distances, volumes, weight... they learn measure
- When they are identifying quantities and patterns they learn numbers
- When they are classifying and ordering, relating and build the abstract thinking
- ...

Since birth up to 8 years, children build on the numbering up to 999. Numbering is a language that can't be learned without the abstract thinking and mastery of space orientation. That is why we say: "There are not small mathematics", there are important and complex mathematics at this level.

Children always learn mathematics. That this learning be more solid only depends on increasing the quantity and quality of the learning situations that are found

#### How can we support this goal starting from the museum?

The Museum shows a very suitable attitude to support this goal when it says, "We don't teach, but they learn" "We only make self- experience possible". This attitude is indispensable for good intervention.

Other important things you can do are:

• Showing models of a wide variety of learning situations and materials to present them. From the museum, activities and materials can be disseminated that enlarge

the scope of mathematics beyond the numbers, showing for example puzzles, patterns, logical processes, constructions, games...

- Helping to identify mathematical thinking in experimentation and in everyday situations. It is often difficult to recognize mathematical thinking in the activity of children of these ages and it is important to be conscience of when there is one.
- Providing tools to interpret the actions of children and to make them evolve. Now we have the description of the Learning Trajectories [2] in these ages based on the research. A tool that will greatly help in the interpretation and improvement of learning
- In accordance with the idea that the child is the protagonist of his learning. We only propose situations, materials and specific interventions that help him to learn. We say. "There are not small mathematicians", there are people who learn the math they need for their age



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# CG3: Hands-on and virtual hands-on

By M. Alsina (editor)

«Is virtual virtuous?»

The presence of technology in all social areas, school, home, relationships, information... is continually increasing and gaining new competencies. The math component is enormous and not always evident. The proposal was to discuss which and how much technology is needed in the museum, but defining the purpose too.

Ana Cristina Oliveira, who was the moderator of the conversation group, took the opportunity to present *Atractor*, a Portuguese association offering a wide range of resources. See next paper, and a brief description in the outreach part of this volume.

Daniel Ramos invited participants to answer the questions: Should we use computer exhibits and new technologies in our exhibitions? When? How much? He was interested in bringing up some cases of digital exhibits in museums and exhibitions, as a sample of different uses, approaches and conceptions of public outreach. He proposed to discuss advantages and inconveniences, tips to add (or remove) technology, exchanging their visions on the topic. Anyone who ever faced this dilemma, was invited to share solutions and concerns!

In the session, Jose Luis Rodriguez presented the software *NeoTrie VR* and its potential for virtual reality exhibitions in museums, science fairs, etc. That software allows to select experiences and activities that can't be done with manipulative materials or other digital supports. More information can be found at: http://virtualdor.com/es/NeoTrie-VR/

Paul Stephenson focused on the goal to design software encouraging collaboration. The reason is coming from the comparation between children grouped around a physical exhibit and a digital one. At the physical exhibit they collaborate and communicate spontaneously. They may do so around a computer screen however, since their normal interaction with a such a device is private, this sharing needs to be encouraged. The question was: what can software designers, teachers, explainers do to help? He shared out real geoboards and invited to bring up the corresponding interactivity, nrich.maths.org/2883, on your own device and compare the experience.

Diego Lieban contributed from his experience, since they are exploring some connections with physical and digital resources for STEAM Education. Their focus is in manipulatives that foster mathematical explorations, either to favor spatial reasoning or to open questions and discussions from their use or their modeling process. Most of their activities are developed through GeoGebra and Tinkercad due to their intuitive interface for educational purposes and friendly connections with 3D printers. Also, they promote student-centered learning approaches where students are encouraged to adapt and create their own materials and they are learning math from these experiences.

# Is virtual virtuous?

By Ana Cristina Oliveira (Association Atractor)

**Abstract:** In this paper I address some of the advantages and inconveniences of virtual exhibits versus physical ones. To illustrate the guiding topics of this discussion I present three physical exhibits, belonging to Atractor Association (a non-profit Portuguese association whose aim is to raise the awareness to Mathematics), and the corresponding virtual versions. Accordingly, I will draw one main conclusion: both approaches are beneficial and complement each other. At the end of the paper, I dwell on other issues related to this subject, as "Is this discussion more pertinent in Mathematics?" or "Will virtual exhibitions lead people to feel discouraged to visit physical ones?".

Being invited to moderate the Conversation Group "Is virtual virtuous?" of the Matrix Conference 2018 gave me the opportunity to reflect further and put into writing some considerations, mainly based on my own experience, on a topic which I consider relevant and interesting: the advantages and disadvantages of virtual exhibits versus physical ones.

The importance of physical exhibits is undeniable: they allow the public to handle or have a direct interaction with the original objects, which can lead to a longer effect on the visitors. In addition, real objects are undeniable, and therefore absolutely credible, unlike the virtual part which often causes doubts related to the possible existence of some kind of staging or devious trick. Even on the social part, there are advantages in using physical exhibits: many persons visit exhibitions in pairs or groups, which helps the sharing of ideas; contrary to the virtual part, which tends to be more solitary. However, we live in an era where the reliance on technology is continually increasing and should not be neglected. Furthermore, there are some aspects where virtual exhibits can do what physical exhibits cannot (the reverse is also true): they allow an extension of the physical exhibit; create experiences that are not physically feasible (or would be extremely difficult to build); save production costs; solve conservation/preservation problems; reach people and institutions that otherwise couldn't access the exhibits, ensure an accuracy which is essential in mathematical models and hardly attainable through a physical realization, etc.

So, how can we deal with the dilemma: virtual exhibits or physical ones?

My opinion on the subject is that, whenever possible, we shouldn't face a virtual exhibit as a replacement of a physical one: instead, the two approaches complement each other. To illustrate my point of view, I'll describe two exhibits developed by Atractor (www.atractor.pt) – a Portuguese non-profit association devoted to raising the awareness to Mathematics.

In order to celebrate the World Year of Mathematics, Atractor launched in 2000 a large exhibition (which received more than two million visitors) entitled Matemática Viva (Maths

Alive), in Pavilhão do Conhecimento, Lisbon. The Hyperbolic Slit (Image 1) was probably one of the exhibits with greatest impact, both by its size and by the surprise caused on the visitors [2]. It consists of a large metal plate with a hyperbolic slit and also of two inclined metal rods that rotate on a movable disk located on the bottom of the exhibit. Although the rods appear to be in collision with the plate, this never occurs.

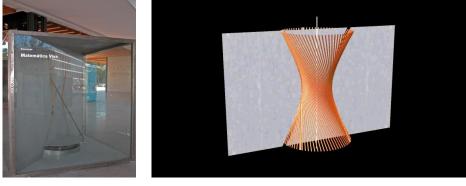


Image 1

Image 2

Hardly any virtual version of this exhibit could have an impact, in the public, similar to the physical one (now in University of Porto). Actually, even nowadays, for those who have the opportunity to pass near this model, it is common to overhear discussions and sharing of ideas of groups of visitors in front of the Hyperbolic Slit. However, like all exhibits, we may find some limitations: on the one hand, it is very difficult to transport and show it in other locations, and, on the other hand, for those who have never thought about the subject, it may seem elusive and most likely is too hard to perceive the geometry involved. These problems can be overcome by using Atractor's virtual replica of this exhibit [3], in which the user can understand how it works, and can also build a large number of new virtual versions of the model, by changing the angle of the rod and/or the distance to the axis (Image 2).



Similar considerations are applicable to other Atractor's physical exhibits, namely the Conic Billiards (Image 3): based on a game of great popularity – billiards – conventional billiard tables were adapted, in order to illustrate the reflective properties of the three conics. Once again, the great impact that this exhibit had on the public, as well as the fact that visitors could attest "in loco" and physically "the reflection properties" of the balls after hitting the conic boundary of the billiard table, makes this physical version irreplaceable

by a virtual one. However, the same limitations of the Hyperbolic Slit can be pointed out: 1) difficult transportation; 2) necessity to clarify some mathematical issues related to the exhibit. Such limitations are overcome once again by a virtual extension of the exhibit created by Atractor [4], in which the user can not only make some simulations as well as perform other experiences which are, in practice, infeasible (Image 4).

So far, we have only considered mathematical exhibits, but similar questions can be addressed in other areas: is the discussion "Is virtual virtuous?" more pertinent in Mathematics? Although some of our considerations are certainly valid for other areas, the abstract nature of Mathematics leads to specific difficulties. For instance, the need for finding a style that is both rigorous and clear for a general audience. Moreover, Mathematics usually demands an individual endeavor to be understood, and this may be easier to be achieved through a guided virtual interactive model. In addition, some mathematical ideas and results are very hard to convey by physical means, but perfectly adapted to be told virtually. Consider, for example, the following question: What would happen if we closed the kaleidoscope of the cube? What would we see if we added a fourth mirror, closing the kaleidoscope? (Image 5) Technically this is a question of difficult resolution: we needed to be inside the kaleidoscope to see what is happening. But virtually the answer can be much easier, and it is explored in the following movie from Atractor: [5].

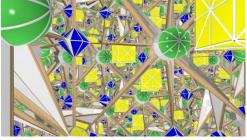


Image 5

Some may claim that, by providing a virtual exhibit in addition to a physical one, people may feel discouraged to visit museums, as they have a cheaper access to the contents over the internet. Although this may happen sometimes, we notice that the inverse situation may also occur: visiting virtual exhibits may trigger interest in knowing the physical ones.

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- [5] https://www.youtube.com/watch?v=Ez4ukA8AWj4

# What's the difference?

By Paul Stephenson (The Magic Mathworks Travelling Circus)

If I want to learn to ski, I can enter a cubicle, set my feet on pressure plates, grasp would-be ski sticks, put on a headset and practise the necessary skills in a virtual environment. When I emerge blinking into the light, I may have to adjust to factors I hadn't considered but, on some basic level, I shall be able to ski.

What about experiencing mathematics?

Mathematics is abstract by its nature. You only really experience Mathematics – with a capital "m" – in your head. But for most students, most of the time, mathematics has a small "m"; it exists in a world shared with fellow students and teachers: it is a communal activity. Vygotsky had a special word for the whole environment in which a learner exists, which translators usually transliterate but leave in the original Russian: *Perezhivenie*.

When children are grouped around a physical exhibit, they collaborate and communicate spontaneously. They may do so around a computer screen but, since their normal interaction with such a device is private, this sharing needs to be encouraged. What can software designers, teachers, explainers do to help?

I encouraged readers to do an identical activity with a real geoboard and a virtual one, in fact nrich.maths.org/2883. Mike Pearson, who designed this interactivity, did the best he could to give us the illusion that we are physically looping the rubber bands round the pegs.

Of course, my aim was to pose the question: "What's the difference?". I suspect there is no simple answer to this question but readers are invited to compare their own physical, affective and social responses to the two experiences.

# CG4: International Cooperation of Maths Festivals

By Eoin Gill

«Let's do something together»

There was a broad representation of people present, representing different Maths popularisation initiatives. There were also diverse nationalities represented including Catalonia, Spain, US, UK, Portugal, Serbia, Germany, Romania.

The conversation group was led by Eoin Gill of Maths Week Ireland who outlined the aim of the session was to explore possibilities for international cooperation between Maths Festivals.

The session firstly would discuss the variety of events happening and then discuss how cooperation may be helpful.

To initiate the conversation a number of initiatives connected with people present at Matrix were presented. A short video was presented of May Month of Mathematics Festival Serbia to show a variety of activities possible in a maths festival.

#### https://youtu.be/hM8BTE5J3Vc?t=128

Following this, each delegate was asked to introduce themselves and their organisation/activity and to indicate why they chose this Conversation Group and what they hoped to get out of it.

The following is a short summary of the introduction of participants:

• *Nuria Fagella*: Is a professor in University of Barcelona involved with maths teacher education. She organises a Maths festival in which her undergraduate students prepare a fair for high school students. With support from MMACA

• *Tim Nissen*: Is chief designer with MoMaths the National Museum of Mathematics in US. They organise a successful outdoor maths festival in New York.

• *Paul Nugent*: Is with Maths Week Ireland and is a teacher who is experienced with various maths and physics outdoor displays. He was hoping to learn what is going on in other countries.

• *Guido Ramellini, Enric Braso & David Pinyol Gras*: From MMACA. They added several initiatives happening across Spain. Maths in the Street Madrid, in Galicia there is a new area in the museum for maths. In MMACA they celebrate Martin Gardner Day and also International School Day. They suggested that initiatives could be connected together in a common project and that a network could be useful to connect people. Another model he

highlighted was Global Maths Week, which although was mainly taking place in schools was something people could become involved in.

• *Sheila Donegan* & *Cordula Weiss*: Involved in organising Maths Week, were hoping to learn what other people were doing.

• *Katie Oldfield*: Joined Maths Week Scotland this year, the second of which ran in September. It started mainly with schools and she worked to expand it to adult audiences. Also plans to develop a year round programme to promote maths.

• *Tiago Hirth*: Self-confessed "maths enthusiast". Works for the National Museum of Natural History and Science, Lisbon which has maths displays and is part of the Mathematical Circus. He organises G4G Celebration of Mind, Maths Fairs and other events.

• *Aleksandra Ravas, Jovan Knezevic & Tijana Markovic*: Would like to cooperate in international projects. They also felt that there is a need to create recognition of the importance of maths festivals.

• *Rogerio Martins*: Has been involved in various maths popularisation initiatives, including television programmes. Suggested that in Portugal there is not a great tradition of science fairs.

• *Aaron Montag*: From TU Munich, has had some involvement in STEM outreach but would like to be more involved in Maths Promotion.

• *Jordi Deulofeu*: Is involved in teacher training in Barcelona. Would like to be able to involve more people.

• *Pepus Daunis i Estadella*: Teaches Statistics and involved in local science and maths festivals.

• *Paul Stephenson*: Has for many years run a travelling exhibition which he brings to schools and festivals. Based in UK and Germany. He was involved in Maths Week Scotland this year. Has been very impressed that MWI & MWS have been able to get schools doing their own events to be part of these festivals. In Germany he has been training senior school pupils to present to junior schools.

• James Grime: A professional maths presenter, he is interested in helping to promote maths.

• *Irina Vasileson*: Representing the European Commission's Scientix.eu. From Romania, where they have a tradition of contests in maths, maths camps but not a maths festival.

• *Tom Crawford*: Involved in establishing the Oxford Maths Festival and is a professional maths presenter. Would like to find out what is going on elsewhere.

• *Paula Hamilton*: Had a career in risk management in banking in the US, now runs a business to promote maths. Is particularly interested in gender balance.

After the introduction of some (many) initiatives a slide was presented of a sample of other Maths Festivals from around the world. It was emphasised that this was not an exhaustive

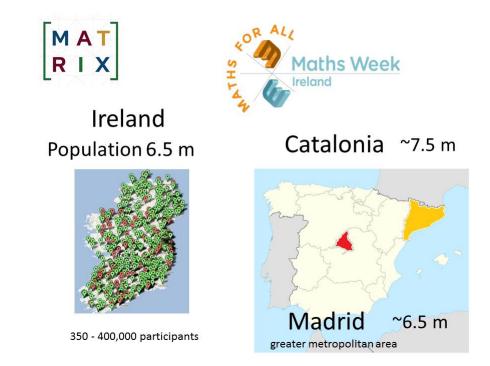
list but would show examples of other activity that may be inspiring.

- Maths Jam: is a movement of Maths Enthusiasts. Originated in UK with people such as Colin Wright and Matt Parker, has spread around the world. Is for people already interested in maths and could be part of a maths festival if links with a local Maths Jam, or a Maths Jam could be established for the festival. https://www.mathsjam.com/
- Julia Robinson Maths Festival: is a US initiative that has run in many countries. The foundation supports locally organised after-schools events to encourage students to think critically. Typically activities are laid out on tables and participants are free to go to any activity and work in teams to solve challenges and also to move when they wish. A maths student/teacher or professional volunteer manages each activity. http://jrmf.org/
- **The New Zealand Maths Week:** is an online activity for schools across New Zealand. https://www.mathsweek.co.nz/
- **National Math Festival:** is a day-long event in Washington DC. https://www.nationalmathfestival.org/
- **G4G Celebration of Mind:** are meetings or events to celebrate Martin Gardner's birthday (21 October). Martin Gardner was an American populariser of maths, science and promoter of scepticism. The Gathering4Gardner Foundation co-ordinate these events and have resources and advice on their website to run events. https://www.celebrationofmind.org/
- Mathematics and Statistics Awareness Month USA: this is coordinated by American Mathematics Society and takes place across the US every April. http://www.ams.org/mathstatmonth/msamhome
- **Pi Day:** takes place on 14 March (3.14) each year. It is marked by schools mainly to do fun maths activities. https://www.piday.org/
- **Global Math Project:** was established by James Tanton (Mathematician in Residence MAA). The idea was to get as many people around the world doing the same maths activity roughly at the same time. An ingenious activity was devised by Tanton called Exploding Dots. The Global Math Project has also billed itself Global Math Week and starts on the 10/10. https://www.globalmathproject.org/

Following the presentation of these initiatives a video of Maths Week Ireland 2017 was shown. **Maths Week Ireland** was established in 2006 by the Calmast STEM Outreach Centre at Waterford Institute of Technology. It is a partnership of higher education and other groups conderned with the promotion of maths. It runs every October in the week containing the 16th and includes both weekends. The festival includes Northern Ireland and the Republic of Ireland and begins with Maths in the City in Dublin and the following weekend this event takes place in Belfast. Universities, visitor centres and other groups present events for schools and the public and some key events are organised centrally. Schools are encouraged to do activities in-school; activities are suggested on Maths Week website and many schools

organise their own activities. Teachers are encouraged to register on the www.mathsweek.ie to show how many pupils will be involved. In 2018, 354,000 were recorded. At the time of the Matrix meeting it was estimated that up to 400,000 could have participated in Maths Week 2018.

The organisers believe **the key to success is partnership and the enthusiasm of teachers**. It is important that those involved are free to organise events that suit them and to have ownership of those events.



The conversation that followed included the following contributions.

Katie Oldfield said that with Maths Week Scotland, working in partnerships has been very important. Schools did great things themselves. Many people were keen to do things but didn't know how to go about it. The government is providing some funding for people to do things. She is working on how to connect people with ideas and provide a clear path from idea stage to running an event.

Tiago Hirth asked how they get partners to put in money.

Katie explained that partners such as mathematical societies and other that want to promote maths gave money along with the government. They didn't need to seek outside funding.

Eoin Gill said that in terms of what he outlined about Ireland, the partners were groups that wanted to promote maths and they were expected to provide resources themselves. This could be as simple as providing a venue. They may design and deliver their own events but Maths Week organisers will also provide them with a presenter if they wish. The main funding comes from Government in the Republic and Northern Ireland. The main items of expenditure are design, printing and postage of a range of posters to all schools (c. 5,000), media & PR, presenters fees, travel and accommodation, project management, website and resource development.

Tim Nissen asked how do you persuade partners that it is worthwhile and do festivals contribute to "real math".

Paul Nugent said that when it comes to Ireland it was a case in the early years of finding the right person in each institute but once they participated they could see the impact. New partners heard about it from existing partners. It is so big now that organisations would feel embarrassed not to be involved.

Tim Nissen added that presenters were their best advocates.

Guido Ramellini said that there are many different things that can be done that can be shared. For instance in Maths Week Ireland 2017 all the schools and venues were closed because of the hurricane. The presenters were confined to a hotel and set about broadcasting short demonstrations and mathematical activities via Facebook. It was not expensive. Much can be done this way creating videos and media and shared. Also the EU could become an umbrella for maths promotion. More mature structures should help newer events. Could look for EU resources to help emerging events. Everybody can start with one activity.

Irina Vasileson suggested that maths festivals and outreach groups could make use of SCIENTIX.EU. This is a European Commission initiative established in 2011 to support teachers of STEM. It is headquartered in Brussels but is a network across Europe. There are national points of contact in most countries. It provides workshops for projects, webinars and conferences. They also have a future classroom and a conference room.

Tiago Hirth asked about typology of events. He organised events for European Researchers Night and found that the talks often did not work well. Were there types of events that worked well? He also found that teachers did not like to bring their pupils to a fair where they could walk around different activities rather than have a structured session.

Eoin Gill suggested that the fair model could be structured to the teacher's liking if there was a set schedule for groups to participate in each event. Often events can be tweaked to suit particular audiences.

Paul Stephenson explained that Magic MathsWorks goes to schools and sets up in an open space (not an auditorium) with related activities and the format works well. It works very well for families and a very good atmosphere develops. The activities tend to be open ended.

Katie Oldfield related a sucessful event at Maths Week Scotland that took place in a games café. It was led by a researcher in game theory and there were many standard games laid out for people to play. The researcher was able to move about and talk one on one to visitors.

Guido Ramellini highlighted the work of Imaginary and suggested that their platform may be of use. He pointed out that the next Imaginary Conference will be in December in Uruguay and perhaps could be used to link with initiatives in South America.

Eoin Gill then presented a slide asking what challenges we face.

Eoin Gill said that for Maths Week Ireland it was much easier to engage schools that adults/ public. It would be good to travel to other festivals to see how others do things.

Aleksandra Ravas said that development of events depends on the kind of audience you want to engage with.

Tiago Hirth added that accessibility is a key factor. It is no good organising events in areas where there are poor transport links. He also asked if maths events had private sponsorship.

Paula Hamilton talked about private partnerships. She said that code.org had large funding from Google and Pixar and have entered a large number of classrooms.

Eoin Gill observed that in Ireland there seems to be a big interest by the tech companies in promoting coding. Perhaps we should look at working together to persuade them that there can be no coding or technology without maths.

Alecksanra pointed out that focus shouldn't be on traditional media, that few young peope read a newspaper. Social media is very important to reach audiences.

In summing up, Eoin Gill said that there is a diverse range of festivals and activities. He felt that the Maths Week model has been very successful and should work in other countries. He was unsure of the influence of scale but that it might work on a city, regional or national scale. He was of the opinion that a week was the optimal time, a day was too short to allow everybody to get involved and that a month was too diffuse and diluted teh impact. Partnership was the key. A diverse offering of events and activities was important for access and to have "Maths for All".

# CG5: On Teachers training at Museums

By Montserrat Alsina i Aubach (Dept. Matemàtiques, Universitat Politècnica de Catalunya)

«Don't be so formal!»

#### Main question

The initial and ongoing training of teachers is conducted in different ways in different countries. In fact, education system in each country regulates the training itinerary for teachers, through the corresponding university degrees, perhaps complemented by masters, leading to initial training or accreditation to act as a teacher.

However, given the importance of both training in different competences and ongoing training, it is clear that formal itineraries are not exhaustive and could be complemented by initiatives outside the formal circuit.

From that point, main goal of the conversation group was to discuss which could be the formative role of less formal settings such as museums. Namely: what about teachers training at museums?

#### Expectations

The group included people related with outreach and museums in a wide range: museum directors, museum module designers, exhibition creators, project coordinators, dissemination and outreach professionals, math exhibit guides, university professors and researchers, secondary school teachers, etc. All of them join the group willing to know and share experiences, to exchange ideas and mechanics and, as somebody said **to get inspiration from each other**.

First of all, as a self-introduction, each person tried to put in a word what expects at a museum. Answers were so interesting, offering a kaleidoscopic look, and complementing themselves even some of them sometimes are opposite words: discoveries and re-discoveries, fun, questions, answers, engagement, freedom to create and learning, to learn something new, to learn without taking care, to be impressed/amazed, to change my life, etc. The new word *edutainment* (for education + entertainment) was also used, and somebody described it by saying "first aah" (what means surprise) "and then aha" (what means understood, got it).

#### Experiences and points of view

Participants contributed looking to their own experiences and trying to relate them to teachers training.

People from MMACA were convinced that Science museum/centers can play a very important role in initial and ongoing teachers' training, especially if skills-based education is taken as a reference horizon.

From Manchester Metropolitan University they talked about the open-ended informal learning experiences they are developing to foster and support mathematical expressiveness. See Vinay Kathotia & Ricardo Nemirovsky contribution at this volume. These creative environment can be used also by teachers, to experiment themselves. From MoMath, they support the idea that activities in museums are also interesting for teachers, using as example the logo design activity based on symmetries (with the possibility to get a printed T-shirt). A representative talked about *Imaginary project* underlining some strengths for teachers: introduction of equations, not be afraid of algebraic expressions, easy generalization and production of examples... Also resources like Divermazo or activities like Math festivals (see next part of this volume of proceedings) provide learning benefits for teachers. People from UK described a bit of some current teachers training programs related to museums.

From university point of view, personally involved in "formal" teachers training, it was pointed out that non formal training does not mean "naive" training. Discussion sessions conducted by expert and qualified people can be held in museums and other settings, helping to go deeper in both directions: content and skills learning. In addition, by expanding the learning environment to include museums, a very positive aspect is revealed: mathematics, or science in general, comes close to people, to society. In this sense, museums and similar centers act as true bridges between knowledge and society.

Discussion was focused on what museums have to offer to teachers and how they are more likely to react. It is clear that visiting a museum, teachers can also learn (if interested), can gain experience and expertise in content and skills and they can enjoy feeling as an student discovering new things. But at the same time, they may be afraid their students realise that they (still) don't know all that stuff. Sometimes they feel stressed if they have to answer questions in front of their students. Thus, so they resist putting themselves in the same role as the students, and as a consequence cannot enjoy themselves.

To engage teachers some good practices were suggested. It is important to pay attention to them in order they feel special and some complicity could be developed. Information before the activity can be offered in order they feel more comfortable: if they know the answers to possible questions, their self-confidence increases. That information can be given in different ways: impersonally in writing, in a teachers sessions, etc. Thus, it is important to provide information or didactic materials, and to explain the rules of the activity, but not to set mandatory homework for them. Moreover, some events can be organised just for teachers, to give extra information, tips and tricks,... to simulate students sessions, or simply to allow them to meet and have fun, and make them to feel at home at the museum.

#### Conclusion

In order to get some quick conclusions of the conversation group session, participants also make a list of what a list of what teachers can find in a museum to count as ongoing training.

exchange, openness, new perspectives, open problems, a lot of questions self-confidence, comfort, practice of skills, knowledge, learning pleasure, discoveries, fun, to enjoy mathematics, pleasure teaching math, contagious curiosity epidemic, expressiveness to break down artificial division between cognition/affect, reason/emotion a newfound sense of delight and wonder in mathematics



Looking at that, no excuse, it's certainly a good invitation to go to the museum for "in-formal training".

# CG6: Museums and scientific vocation

By Francesc Rambla i Marigot (Secretariat of Telecommunications, Cybersecurity and Digital Society. Government of Catalonia) *«Mummy, I want to be a scientist!»* 

#### Introduction

The report "Encouraging STEM Studies for the Labour Market" [1], prepared at the request of the Committee on Employment and Social Affairs of the European Parliament in 2015, analises the situation of Labour Market in Europe in STEM related areas. This document states that, in the European Union, employment of STEM skilled labour has increased in spite of the economic crisis, and demand is expected to grow. Because of this growth and of high numbers of STEM workers approaching retirement age, the document forecasts that around 7 million jobs are going to open until 2025. Although the share of STEM university graduates has slightly increased in 15 Member States and at European level since the mid-2000s, a decline in the share of STEM VET graduates is the predominant trend at the national level.

The study "Women active in ICT sector" [2] developed by European Comission in 2013 shows women are still underrepresented in the ICT sector. Out of 1,000 women with a bachelor degree in Europe, only 29 hold a degree in ICT (compared to 95 men) whilst only 4 eventually work in the ICT sector. Despite the evidence which proves that women's access to an ICT career is essential for the sector's long-term growth, there remains a large gender gap in Europe's ICT sector.

What these two reports show is not far from what we see in other places in the world. Concretely, in Catalonia although ICT sector unemployment rates are and had remained low during last crisis (it was nearly a 5% in 2017 while the average was 17,7%), the decline of graduates in ICT-related studies remains.

How can Museums of Mathematics help to achieve a higher rate of STEM graduates? This was the aim of the discussion group and, of course, there's not an easy answer to this question and that's why we are going to make some assumptions.

First of all, let's imagine a group of students who are going to finish primary education. It can be the members of a class, a school or a region, it doesn't matter. In this group, we can create two subgroups: those who are performing well in maths and those who are not. In the context of this assumption, it is not important how this performance is assessed.

Well, given these two subgroups, let us suppose that we ask them if they know what they want to study in the future. Given that we are talking about primary students, it would not

be surprising that only an small fraction of them know it. In any case, it's quite probable that, in this fraction, an small number of them will be choosing STEM-related studies. It is not important in which of previous subgroups they were –good or bad performance in maths–, if they know what they want to study, they will do their best to achieve it.

Now, let's focus on the rest: those who don't already know what studies they are going to choose. Who of them is willing to follow STEM-related studies? Well, we can't obviously know that, but we can assume that students performing better in mathematics will be more likely to study STEM because STEM-related studies use to be intensive in maths. So that our first assumption was:

Students that perform better in mathematics are more likely to study STEM. So that, if we improve overall performance in mathematics, we will increase the number of students in STEM.

What about girls in STEM? The study Role persistence and gender stereotypes challenges in the choosing of higher studies. Teachers and families roles developed by UOC (Open University of Catalonia) 3 tries to identify the how role persistence and gender stereotypes influences in the choosing of higher studies. A remarkable finding of this study is that both girls and boys use to perceive that their skills are worse than what they really are but this difference is higher in girls.

Our second assumption was:

Girls perceive that their skills in technology, science or maths are worse than what they really are. So that, if the perceived skills of girls in these subjects improve, we will increase the number of girls in STEM-related studies

These two assumptions were the starting point of our conversation group. So we launched these questions to the attendance:

How can overall performance in maths be improved? How can the perceived skills in maths of girls be improved? What can museums do (to help us in these)?

#### **Conversation group session**

During the session, participants in the conversation group mostly agreed with the first assumption but not with the second one. Although it is true that girls perceive that their skills in STEM are worse than what they really are with a higher difference than boys, general agreement was that this is not the main cause for girls rejecting STEM careers. Lack of referents and stereotypes seam more feasible causes for it. In fact, both men and women tend to have a worse perception about their skills in STEM related subjects than what they actually are. Although It exists a relevant gender bias in some STEM areas, specially in engineering, it doesn't in others such as chemistry, biology or geology, and they are also intensive in maths. So, probably maths are not more relevant in the choice to girls than they are to boys. One of the overall goals of Mathematics Museus is making maths more attractive and easy to understand. In the Momath website we can read that "its dynamic exhibits, gallery, and programs are designed to stimulate inquiry, spark curiosity, and reveal the wonders of mathematics". The MMACA website states that one of the objectives of MMACA association is: "disseminate and estimulate a positive social image of mathematics: by making it closer to people through interactive experiences and manipulative activities, and by showing its presence and the role they play in our culture and social progress". So that making mathematics more understandable is in the essence of Museums of Mathematics.

## Conclusions

Is all that enough for students to perform better? Certainly it is not because the Museum experiences are punctual and isolated, while something recurrent and constant would be required to improve the process of learning mathematics. In any case, some ideas arose during the session:

- Training teachers to use the museum strategies at classroom;
- Producing activities that can be reproduced outside the museum.

And, what can be done to address the gender bias in some studies? Some ideas also arose:

- Including gender perspective in museum educators training in order that manipulative activities in the museum are performed both by girls and boys;
- Changing scripts used by museum instructors to explain the activities to incorporate gender perspective;
- Adding women who are referent of Mathematics in the exhibition area to break gender stereotypes.

## References

[1] Encouraging STEM studies for the labour market.

https:

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[2] Women active in the ICT sector.

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# **CG7: Statistics**

By Pepus Daunis-i-Estadella (Societat Catalana d'Estadística - Universitat de Girona)

«In the museum and for the museum»

## Introduction

In this conversation group "In the museum and for the museum (Statistics)" there were different participants representing nationalities such as Catalonia, Spain, US, Serbia, Brazil. The people who had interest and joined to the conversation group were: Aleksandra Ravas, Alfonso Peres Osia, Aniura Milanés Barrientos, Daniel Sierra, Erika Berenice Roldan Roa, Fernando Blasco, Goretti Torrent Tort, Joan Alemany Flos, Judit Cucala, M. Mercè Saavedra Gandia, Pedro Latorre García, Pelegri Viader, Marina Brasó Vives, Tijana Markovic, Guido Ramellini, Enric Brasó, Sergio Belmonte, Pepus Daunis-i-Estadella.

The conversation group was managed by Pepus Daunis-i-Estadella of Societat Catalana Estadística and Universitat de Girona who outlined that the aim of the session was to discuss and identify problems about including statistics and probability in exhibitions, fairs...

The session firstly starts stating that statistics (and probability) is a part of mathematics that is more present in the social reality than within museums.

To initiate the conversation two questions were raised to people present at Matrix: Can we transfer the social reality of use of Statistics to a museum module? How can we deal with uncertainty?

## Contributions

The following is a short summary of contributions.

A first experience was presented from Serbia. A math festival including probability and statistics. Posters with 13 short stories about probability that show this interesting branch of mathematics in a clear and simple way.

https://www.flickr.com/photos/cpnsrbija/albums/72157681594795063/

A second contribution from Brazil stated that there weren't so many activities that cover statistics.

MMACA people pointed out that from the very beginning of the MMACA project, it became clear that their desire to show manipulative forms related to statistics and probability clashed with the difficulty of finding materials and dynamics compatible with the times of attention and implementation of an exhibition, and with an assumable cost. Thus, the specific hall of their exhibition often shows the modalities of a guided workshop. See next contribution by G. Ramellini in this proceedings.

Another contribution from the perspective of secondary school, asked the audience what is the problem? Exhibits, contents...?

A representative of science museum Cosmocaixa stated that they have scheduled 4 or 5 activities and conferences on probability and statistics that have greatly satisfied the audience. One of the fundamental concepts underlying communication of maths are shocking and attracting titles.

From the Universidad Politécnica de Madrid stated that there is a lack of knowledge of statistics in Spain. This implies an added difficulty in introducing, using, and attracting people towards statistics. This opinion was shared for more participants.

On the other side, from the society of mathematics and statistics of Aragon, they brought some experiences about simulation. Some of those simulations can be found in:

http://conexionmatematica.catedu.es/azar/

Among them, the typical problem of the Birthday Paradox and the chance of two people having the same birthday, rolling non-symmetric dices...

A brief demonstration was carried out.

At this point the use of computer and simulation tools arose. Is it good or not? Is it an essential tool?

A bundle of experiences raised out, without the use of computer:

- Collect data throwing balls to a basket
- Experiences with characteristics of participants
- Collect data at museum with coloured stickers marking rooms
- Estimating the total number of fishes catching and tagging fishes and refishing some of them.
- Estimating the total number of taxis observing taxi numbers
- ...

But with computer are also possible more complex strategies like generating data through a personal chip (instead of coloured stickers). Computer systems can collect and generate data with the information provided by the chip: how many rooms have been visited, which is the room where the people expend more time... One of the ideas was also try to make some conjecture about data in first place, then to take a sample in order to verify or refuse it. Perhaps this strategy can increase the interest of people about statistics.

Some traditional experimental tools are also reminded: the Galton machine with different sizes, materials and formats. Probability games with amazing coincidences, as the experiment with 8 balls randomly placed on 15 holes, checking how some balls have coincided in the same hole. Or about the non-regularity of randomness.

## Conclusions

In closing remarks people said that perhaps those are good activities to make conjectures about.

This conversation group stated that there are lots of things to do. Shared resources and experiences like this Matrix conversation group can help to improve work with statistics.

A final note is that is possible to use computers but without abuse. A use of computers can help to deal with statistics and probability but in a comprehensive way.

# **Statistical Exhibits in Science Centers**

By Guido Ramellini (MMACA)

Statistical (and probability) exhibits are not so popular in permanent mathematics exhibitions and in Science Centers.

It is possible that there are ideological resistances, linked to preserving a sense of purity of mathematics, perhaps misplaced. Maybe the reasons are more practical: statistics is based on a considerable volume of data, which involves a lot of time to collect, read and reason on it. Meanwhile, the activities of an exhibition demand agile dynamics and fast times.

These are problems that could be solved partly by resorting to technology, increasing the cost of facilities, installations, maintenance, custody, management ...

A good example was the great and successful Big Bang Data exhibition [1] we could visit in Barcelona at the CCCB (May-November 2014).

Moreover, if our museological model is based on the manipulation of objects, it becomes even more complicated.

The fact remains that statistics represent an important bridge between scientific disciplines and an instrument, not only practical, to understand complex phenomena, from climate to biological systems, from the financial market to ecological equilibrium.

It should be added that in the school curriculum a very limited space is left to statistics, which is treated in a hurry the last days of school, with the risk of dealing with flat, uninteresting and current examples or of the schematicity of the formulas.

The MMACA decided from the beginning to reserve a space for these themes, focusing its action on basic elements, adapting different materials and changing, when the public's interest requires it, even the dynamics of the management of visits.

Our educators are authorized to guide a little more a visit to this room and to insist on some more theoretical aspects (confidence interval, calculation of the probability of an event, method of data collection, ...) that would escape the fast and not very reflective fruition of many users.

It is the only modality we have found so far to improve the rating of this room, which remains the lowest of the exhibition. It is possible that sooner or later we are forced to accompany the hand-on exhibits with some computer screens, where to simulate the phenomena and expand the field of observation, as we already do in our workshops.



## Some exhibits of the permanent exhibition of MMACA

## References

- [1] http://www.cccb.org/en/exhibitions/file/big-bang-data/45167
- [2] https://mmaca.cat/

# CG8: Math, Museum and People Dissemination

By M. Alsina (editor)

«Go wider, go deeper!»

In recent years, mathematical dissemination shows a greater social presence. Books, articles and interviews in newspapers and magazines, collections of games, some movies, fleeting but repeated television appearances... are more frequent.

In that context:

can we take advantage of it to be able to impose a more realistic view of mathematics and mathematicians, beyond the easy stereotypes that persecute us?

This debate, moderated by Jordi Deulofeu, attracted many participants who briefly explained their role in activities of mathematics addressed to a general audience. Some of the activities can be found at next part of these proceeding, devoted to outreach activities (Atractor, Chile festival and more).

Most of participants agreed that we are not happy with commonly used mathematical stereotypes; in fact everybody remember some example of influent people proud to say publicly "I'm bad in Math". It seems hard to change this from teachers position, but something can be done from different outreach activities people are involved with. Examples as to use mathematics as real problem solving, or to complement museum exhibits with Math hands-on activities were given. In particular, referring to the role of mass media, there were two significative contributions related to video shows.

Tom Crawford presented his experience from the BBC show as the *Naked Mathematician* and his use of social media (see https://twitter.com/tomrocksmaths as tools for increasing engagement with maths. From Titanic to Arquimedes principle, by using Pokemon or asking about electricity, there is a way to show that mathematics are relevant to every day life, but it does not mean they need to be necessarily serious and boring. This is one point to take in account.

Rogério Martins also shared his experience as author and presenter of the TVShow *Isto é matemática* (cf. www.youtube.com/istoematematica) that's featured in the main Portuguese news channel. This TV show is on its 11th season, has been widely internationally awarded, and shows the connections of mathematics to almost everything: from animal skin patterns, Drake's formula, to the circadian rhythm. Thus the relationship with the real world is another point to take in account.

The session was too short to be able to work out real proposals, but the discussion was very interesting. To sum up, a couple of remarks are outlined to be helpful for incoming

proposals. When people talk about mathematics usually restrict themselves to talk about math teaching or learning, that is to school environment; this is not the same when people talk about economics or history, for example. Thus, to relate mathematics with usual life could be helpful. They can not live only at school. Another remark was that people feel stress when are exposed to mathematics, because a correct answer seems to be always required. Sometimes people just need to enjoy. As example, the series Big Bang Theory changed the way people see scientists, but it is entertainment series.

# **CG9: In E-Motion**

By Maurici Carbó

«Movement and emotion stimulate creActivity»

Since many years ago, there are inspiring interactive math webpages dedicated to mathematics, like Illuminations, that give an attractive insight into mathematics, and many interactive simulations made with Geogebra. All of them were made to be used with computers, workstations or laptops.

As far as I can see, exhibitions in Math museums are a trusted experience. The materials, well explained by human assistants, are very attractive for the visitors. But smartphones are even more widespreaded around the world.

In the museums of mathematics, contrary to usual museums, it is mandatory to touch the exposed materials.

The capabilities of smartphones are closer to reality than the web pages. We can get an interaction by touch on the screen, with one or more fingers, touching, dragging or pinching. It is also possible to interact with images taken through the camera but they are still very far from the real experience, without the possibility of actually touching or weighing anything.

Educational apps in shops, are full of emulated calculators, and instruments with the clear intention of to help solve homework, or to reproduce textbooks, or to improve skills. But the materials that I would like to talk about here, are those that do not seek to provide solutions to problems, nor seek to facilitate results, nor say what is right and wrong, or to test people. Nor to facilitate calculations to solve homework. Here we search for a virtual equivalent to the Math Museums materials.

Some people think in how to make that these small devices can get to interest people in the field of mathematics. These devices needs to show software with very simple and clear interfaces.

There are many ways to accomplish that:

- Showing simulations to explore mathematical objects (Fractals, Graphs, Polygons, ...)
- Little workshops for build, check, visualize and interact and verify theorems (Pythagoras, Hamilton Path, ...)
- Puzzles and riddles. (Wolf Goat Cabbage, Broken Calculators, ...)
- Show mathematical ideas through a transversal approach related to the history of mathematics. (Sumerian Tablets, Egyptian Papyrus, Euclidean Algorithm, ...)
- Attempts to simulate the experience of Montessori materials use. (Integers, Decimals, Fractions, Geoboards, ...)

• Recreations on mathematical objects.

But everything must be very simple to understand at first glance, complemented with videos and images because there are no assistants to guide users here.

I almost tried everything (cf. [1], [2]) with many and sometimes unexpected reactions from the users, but my experience is limitated to Android Devices and from the point of view of a developer. I invite you to share your experiences in the field of Math Apps for smartphones.

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## **CG11: Research and Statistics**

By Montserrat Alsina

«Is the Big Data a Gentle Giant?»

Currently, Big Data is breaking into world of statistics as a real challenge. Actually is a topic of great importance in a wide range of areas, from Sociology or Communication, for example, to Health and Biology, as the moderator Marina Brassó explained from her own experience in this area.

A varied group of participants took part at this conversation group, coming from industry, universities, secondary schools, and museums or other outreach institutions. We all agreed with the challenge related with Big Data, and people contributed adding observations in several directions. By one side related to research specialization area, as Statistics, Number Theory, Cryptography, Coding Theory and others; by the other side related to professional ethics and legal aspects of data privacy, and so on. Thus, we were interested in data anonymization and approaches via removal, encryption or data masking; also in data mining, certain codes as possible solutions...

But, the goal of the conversation group was to discuss how to deal with that important topic at museums and outreach activities:

#### How can we convert this huge landscape into a museum hands-on module? Are we able to use that technology in our benefit?

First obstruction is that little time and attention is devoted to statistics in the school curricula. Second one is that people are still little aware of data privacy problem and the need for protection of information in the different communication channels. This part is not in the school curricula either.

Thus, proposals for first steps in museums could follow these two directions. It is clear that some hands-on modules can be built to increase the interest in Statistics (see also CG7). But referring to information theory and related stuff there is still more to go. Modules, workshops and activities can be very helpful to awake awareness about that.

Time for discussion at the group conversation was not so long, but perhaps some module can be built before next MATRIX conference, because some concrete proposals were already in mind. For sure they will add an extra STEAM value, and will be a good example showing the power of mathematical models.

# CG13: Sharing of exhibits ideas

By Daniel Ramos (IMAGINARY)

«Sharing is caring»

It has been about fifteen years since the opening of the first math museums (Mathematikum, 2002; Il Giardino di Archimede, 2004), and the scene of traveling or permanent math exhibitions is gaining momentum. Today there are more than 50 museums and permanent installations worldwide [1] and dedicated conferences (MATRIX, IMAGINARY Conference). At the same time, many of these agents are embracing open-source philosophies that make easier to re-use exhibits and other materials, reaching greater audiences with cross-collaboration. The session *Sharing is Caring* at MATRIX 2018 served as a measure of the status of the network and as a continued reflection on the open-source paradigm in this context, as started in the Dresden MATRIX conference 2014.

As a test, we gathered a graph of the network relations of the participants in the session. Each participant in the meeting was asked to draw a graph of the interactions between them (as a center node) and institutions which they have collaborated with. Not merely known institutions, but those where some output was created jointly. Then all the participants' input was combined into the graph in Figure 2. This must be considered as just a toy example, since the data is not comprehensive and many important nodes and connections are missing. Nevertheless, we can observe some clusters per geographic regions, and the fact that graph is connected, but not too strongly connected. Some central nodes (MoMath, MMACA, IMAGINARY) have the most connections, but this is most probably due to the fact that these are the organizers of the conference and the conveyor of the session.

The Conversation Group had a discussion about what uses would be most demanded from that network. A quick survey on-site revealed the results in Figure 1. Although the sample of the data is again very small (11 point data), the tendency is clear that most desired use of that network is collaboration projects on content creation (such as European projects and other public-funded calls), Inter-projects where visitors participate in activities of all the partners (such as combined tours or tickets between museums), followed by the organization of conferences. Activities oriented to the network itself (such as catalogs of exhibits, forums, newsletters...) have less priority according to that quick survey.

Finally, a discussion was opened about the open-source paradigm of collaboration, following the ideas grounded in the MATRIX conference in Dresden in 2014. The document called *Dresden Declaration* [2] was a proposal of a code of conduct for the members of the community of math museums and exhibitions, originated from the organizers and some participants of the first MATRIX conference. The preamble states the particularities of math exhibits that make an open source approach possible and desirable:

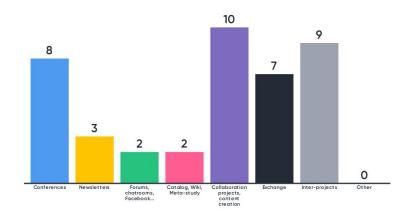


Figure 1: Quick survey: "What would you like to do in the network?"

- *Replicability*. Unlike historical or artistic exhibits, anyone with the appropriate knowledge can reproduce interactive math exhibits.
- *Abstraction vs realization*. Mathematical ideas can have multiple realizations as a museum exhibit. However, design and didactics are essential on a good exhibit.
- *Free knowledge paradigm*. Museums can reflect the openness and universality of math knowledge that is also assumed in research.
- *Community*. Math outreach projects are not competing agents, but cultural services with a common ultimate goal.

The declaration discusses the possibilities to implement open-source paradigms, and then proposes the four core principles for projects that wish to adhere to the declaration:

#### 1. Dissemination of exhibits

It is a main goal of all museums to reach the maximum possible of the population in the most effective way. Exhibits with great design and conveying great ideas are the tools needed to this goal, and preventing its dissemination or keeping exclusivity of ideas goes in detriment of the general public and the philosophy of math communication.

#### 2. Respect to the creators

The designers, creators and hand-crafters of exhibits deserve respect for their work, be it in form of crediting, commercial agreements or other forms. When maths museums replicate math exhibits or otherwise rely on achievements made by third persons, they should within the bounds of possibility give proper credits to the persons who had the idea, developed the concept and realized it.

#### 3. Collaboration

The balance between dissemination of exhibits and the respect for the creators must

be achieved in a framework of mutual collaboration. Exchanging and trading exhibits is encouraged, as well as exchanging experiences on their use. Communication between the various partners plays an essential role.

#### 4. Sharing knowledge

The exhibits are only one part of the value of an exhibition. The knowledge of how to explain these exhibits, how to organize activities, the experiences and feedback from the public and all the immaterial assets that a museum collects is also a high value to be shared with the community.

The declaration is not legally binding and there is no official list of adherents. The Conversation Group did not have the goal of planning any enforcement, but to engage the discussion and measure the degree of acceptance within a small sample group. While only about half of the participants were openly embracing these principles (and many major players of the network were not present), there is a general consensus that having these principles stated and referenced in a fixed document is positive, even for some agents that may be sympathetic to the declaration albeit not being able to implement it by practical reasons (business models or need of copyright protection). The increasing number of open-source initiatives in science communication, and the requirement of many public-funded calls to produce the outputs under open licenses will keep this debate on the table for future discussions.

## References

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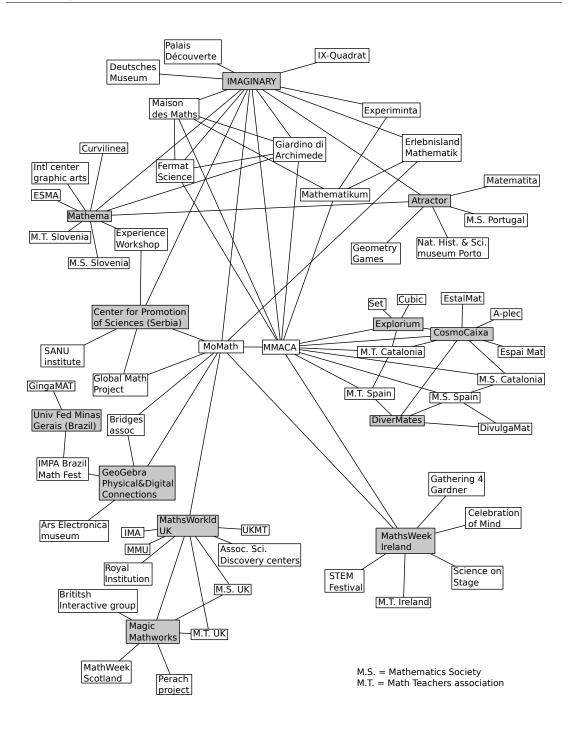


Figure 2: Relationship graph for the participants in the Conversation Group (in grey).

# CG14: Emotion vs Fun: a transforming museum

By M. Alsina (editor)

«I had a lot of fun. I have a lot of questions»

The title of this conversation group, with Cindy Lawrence acting as moderator, was very suggestive, focusing directly to the aim of science museums, and in particular mathematic museums.

The presentation of the discussion stated that the emotional (hearts-on) and playful aspects play an essential role in determining the success of an exhibition, but hide the risk of trivializing content and objectives. Mathematic museums must promote a seductive science, which changes the commonly accepted view of mathematics but keeping this new interest for a long time.

Participants contributed stating experiences and points of view. Some of them are outlined next, suggesting several features to take in account.

Aniura Milanés Barrientos put emphasis on the challenge to hold interest. She works at the Math Department at the Federal University of Minas Gerais in Brazil. She leads there a team of math students that interact with groups of elementary students that visit them, by means of some fun math activities and games. They also organize workshops for math teacher where nice activities and games are discussed. She remarked that one big challenge is always to hold the interest of the children until the math behind the fun can be understood. people agreed it is not so easy.

The setting of Mathematics in a multi-disciplinary museum was also considered as they are relatively open. Katie Oldfield role is to engage people with math in The National Museum of Scotland, a multi-disciplinary museum. There is an opportunity to use the range of collections and areas to showcase maths across all areas of life, beyond it's usual stereotypes. However, she asked where else is that done, and what strategies are there for incorporating maths across a multi-disciplinary museum, in order to share them.

Related to educative system based on schools, Guido Ramellini stated that obviously nowadays the education system requires many different supports, but in any case it is essential that science museum collaboration is always based on complementing its work, using the assets of the language that belong to the museum. To say in a sentence: "It works with the school, but not for the school".

Erika Berenice Roldan Roa insisted in the value of questions and engagement. She talked about how important is that students or people engaged in any learning experience (in particular in science and mathematics) should feel comfortable about asking any question. Also, questions asked in a workshop or any other outreach activity can (should?) guide the flow and content of the activity.

In a more general context, Guido Ramellini pointed out that a Transforming Science Museum must not be obsessed by the number of visitors; its goal is the enlightenment not just the entertainment. Again in a sentence: "It is not trying to put a lot of people in the museum, but a lot of the museum in people".

To sum up, a common point is that the participation and complicity of users must be fully encouraged.

# From fun science to seductive science

By Pere Viladot (Natural Science Museum, Barcelona), Erik Stengler (University of the West of England, Bristol) and Guillermo Fernandez (Independent consultant for museums and science exhibitions)

An opinion paper warning science centres and museums against the shortcomings of "fun science" and arguing for a "seductive science" alternative.

Science centres and museums have undergone a great evolution in recent decades although it seems that, lately, the science museum model has been somewhat stagnant. Since the radical changes of the mid-twentieth century, it has developed towards strategies in which visitor numbers take precedence over other considerations. Alongside a school science that could be described as "boring", a trend has emerged with a focus on "fun science" in museums, hoping to address current shortcomings. In this article we question this view and propose the idea of "seductive science" as an alternative to achieve long-term impact of museum visits.

# The "fun science" trend

In recent times, more and more science centres and museums are aligning themselves with the trend of presenting the visitor experience mostly as "fun", thus identifying the visit with a playful activity. A museum visit must certainly be unique and stimulating, but such an explicit identification with fun-related aspects can, in our view, leave out of the picture the wealth of other elements that a visit to a science centre or museum has to offer. Let us first have a look at various factors that may have contributed to this trend: - The focus on visitor numbers as a measure of success. It is indeed surprising that this is actually taking place in institutions that are meant to show how science works, with visitor numbers becoming, in practice, the only performance indicator of science centres and museums. Mission and vision statements always include a strong societal dimension, such as promoting uptake of science careers (Wotton 2013). Naturally this should be also an important part of the evaluation of success, but we all know how scarce and difficult to obtain such evaluation data are (see for example Cavell & White (2010) for one of the very few longitudinal studies available). As a consequence, there is the risk of just abandoning in practice the role of socio-cultural leadership science centres and museums can have within their communities and replace it with a focus on activities aimed at attracting ever growing visitor numbers. This is often done without the realm of museographic language, sometimes even under the disguise of bold experimenting with avant-garde museology. - The use of business-style market studies. Institutions with a strong societal focus can certainly use market studies to

gain a deeper knowledge of their public and so be able to ascertain what they can offer that is most appropriate. Unfortunately science centres and museums apply such studies in the same way businesses do – in order to learn about public demand and respond to it quickly. Paradoxically, one of the assets of science centres and museums is their ability to offer their audiences experiences previously unknown to them and for which clearly no demand will be detected via a direct and superficial market study. - The identification of science centres and museums with leisure venues. Many members of the public identify science centres and museums as good leisure alternatives for a family day out keeping the children amused, rather than opportunities to share a creative museum experience. Whilst this approach by visitors is certainly welcome, it does not imply that museum managements have to share and cater for it as it is not aligned with the science communication aims and objectives they set themselves. - The influence of "Braniac"-style TV shows. It may seem that science communication works well as a TV product, if one measures by the proliferation of programmes that have some degree of "science" in them, usually through spectacular science demonstrations that are fun and entertaining. Without questioning the good intentions of the producers of such shows, it has to be remembered that their main aim is not to communicate science, but to attract audiences measured by means of "shares". - The influence of trends in schools. There is a current trend in schools which is concerned with ensuring -to a worrying degree - that pupils "feel good" and enjoy being in class, with the ulterior aim of preventing them from developing a distaste for learning, as it is proven that learning is strongly influenced by the learner's emotional state (cf. Roth 1980). In this context, the main reason why many teachers take their classes to science centres and museums is for the pupils to have fun with science (Viladot 2012).

In summary, the demand both from school visits and family audiences seem to push science centres and museums to offer fun. Pairing science and fun can, however, bring about some unwanted consequences, as we will discuss in the next section.

## Fun Science or Seductive Science?

Identifying science with fun can constitute a deceiving enticement towards science for the public, and in particular for prospective students of science careers, who constitute one of the main target audiences of a number of science communication channels, including science centres and museums.

The day-to-day work of a scientist hardly qualifies as "fun" if one looks at long lab hours, data analysis, or code programming, to mention but a few examples. A final year project supervised by one of us showed that those pursuing a career in science tend to distance themselves further and further from the concept of "fun" in science as they gather experience, and that there are numerous other adjectives they come up with to define science, such as fascinating, interesting, exciting, or important, for example (Stengler, Lyons & Fernández, 2013).

#### «Assuming that children will only engage with "fun" things is patronizing»

In fact, assuming children will only do things they perceive to be fun could be considered a patronizing attitude towards them. Many children get involved in say, environmental or animal protection activities not because they think they are fun, but because they realize they are important (Lemke, 2006). However, repeatedly assuming they are only interested in fun could end up becoming a self-fulfilling prophecy.

Moreover, there is growing evidence that making science pleasant and fun for student does not go beyond improving their attitude towards science, as there is no correlation with decisions towards science careers, as reported by DeWitt, Archer & Osborne (2014). A recent broad study by Reach Advisors has shown that after a few years the most intense memory of a visit to a contemporary museum is often related to real objects of particular museographic value, even in the case of young visitors (Wilkening 2015).

Focusing on fun during science centre and museum visits also leaves out of the picture educational considerations such as science centres and museums being ideal environments for constructivist, inquiry based learning (Gerber, Cavallo & Marek, 2001; Lelliott, 2013; Murmann & Avraamidou 2014). Moreover, this can even have a backfiring effect in that it reinforces the idea that learning in class in inherently boring, the "fun" being outside the classroom.

Another often overlooked danger of the idea of "fun science" is that it dissuades scientists from getting involved in science communication, especially the most renowned and prestigious ones. In a day and age in which we are making a big effort to persuade the research community to get involved in public activities it is important to ensure they feel comfortable with it, and trivializing their work by portraying it as a show without substance certainly does not help.

# **Towards Seductive Science**

We all know that another word for "fun" in English and other languages is "diversion", in one or other variant. In English the word "diversion" also kept the original Latin meaning of "turning away" from the intended path. This coincidence is a handy illustration of our view that overemphasising "fun" may "divert" or distract from the intended message about science, education or science centres and museums.

As mentioned earlier, there are many other adjectives that can be applied to science and which reflect much better what it represents: fascinating, exciting, thrilling... This is what "seduces" the scientists to make them willing to endure the hard and less gratifying aspects of research. They know that at the end of the process, obtaining results and drawing conclusions is an unmatched intellectual experience.

#### «Fun versus seductive: From diversion to attraction»

"Seducere" means in Latin "to attract" and this is exactly what should be strived to in science centres and museums – and in schools, too, we dare say–: to promote interest for science; to prevent the children's innate curiosity from fading off with time; to show pupils that a museum visit provides more questions rather than answers; to facilitate that excitement becomes fascination. To do so there are some fundamental elements a school visit should feature, which we list here with our experience and research as a basis, and without aiming to be exhaustive.

**Collectively constructed science.** The core of a science centre or museum is the exhibition. It should become the field where students in small groups collect data, where they observe nature, where the most exciting moments of encounter with the object or the phenomenon will take place. These data can then be analysed in the workshop rooms – their labs–, where they share ideas with their fellow students, and construct their own conclusions, which they can then communicate to the other members of their school or family group. Science is a collective human construction and in science centres and museums, there must be a constant interplay of doing, thinking and communicating, just as in real science, and as such, it is not necessary that everyone in the group does everything: there are different roles, and it is not about having done every single task, but rather about having gone through all intellectual stages and having taken part in the generation of new knowledge as a member of a team.

**Science as a story.** First, science needs to be portrayed as a human endeavour in constant change, embedded in culture, particularly in the culture of the visitors. To do so, science must be told as a story, scientific language has to become a narrative that links concepts with personal cultural experiences, almost like turning science into a new humanities discipline. Starting an activity as a story based in the use of different communication systems will help creating an emotional bond that can be referred to throughout its delivery.

**Science in dialogue with other disciplines.** On the other hand, in a museum natural phenomena are presented out of their context. Objects displayed or exhibits that simulate natural processes need educational approaches which redefine their contexts and link them, again, to culture. This cannot be attained if science does not interact continuously with the other disciplines. Science may well be the central axis for a topic, but at the same level as, and in conversation with, other communication systems, the arts, mathematics, etc., so as to incorporate one of the key aspects of any scientific development: creativity.

**Consolidating learning.** The museum is not a classroom, but the museum's assets can be developed to be an invaluable complement to classroom learning. In the science centres or museums we have little knowledge of how the teachers make links between the visit and the curriculum (or how the discussion will go on at home), as our contact time with visitors is brief and fleeting. Yet it is clear to us that, since two thirds of the visitors are not only looking for fun but for learning experiences, too, and 100% of teachers hope that learning will take place during the school visit to the museum (Viladot 2012), we must ensure this actually happens. The only way to achieve this is through a facilitated activity at the end of

the visit in which participants have to apply their learning and the changes in the way they see reality that have taken place since they arrived. As we will mention below, the interplay of different communication systems and interdisciplinary dialogue will be key to this.

**Calm Science.** It easily follows from the previous points that, just like science itself, the whole process cannot be completed in haste. A visit to a science centre or museum has to be relaxed. Not only because science cannot be rushed, but simply because a high level of attention cannot be sustained for long periods of time, and it becomes necessary to alternate between moments of intense stimuli that require high levels of attention and other more relaxed ones that then allow to bring attention back to a high level. This implies that a museum visit, especially a school visit, needs to be as long as possible so as to include the necessary breaks. A whole morning would seem appropriate. But it is not only the delivery of the actual activity that matters, there are other relevant aspects that need to be taken care of: a welcome at reception that is not rushed; moving through the exhibition floor quietly and without running; museum staff talking in a low voice; and everything that contributes to a calm atmosphere. This is radically different from the common scenario in which crowds of children shout and run around, press buttons without paying attention, etc., to the desperation of teachers, parents and museum staff.

All this is certainly not easy. It definitely is not in the absence of a professional education team. It is not if we do not have educators instead of explainers and guides, if we leave these activities in hands of interns without much experience – internships are something quite different– and without resources at their disposal, if we rely on temporary workforces without continuity within roles. Highly knowledgeable educators are needed –visitors hope to meet experts to help them understand (Viladot 2012)–, trained both in the subject matters and in education, and prepared to deal with diverse audiences and adapt any part of the activity to changing audience needs, knowing that it is their only shot with these particular members of the public. We need educators that can cater for very different visitor groups, with radically different needs and unknown expectations.

We know that in the current economic climate, advocating such a working model may seem frivolous, but education is not, and there is a lot at stake. We know that we are not doing particularly well, that citizens do not feel involved in the issues of science and technology, that fewer and fewer young students want to become scientists. We need to act now.

#### «Entertainment is not enough - We need to aim for long term impact»

Otherwise we will certainly contribute to the entertainment of the population, but without effecting any change in how they see science in the long run or in their ability to recognise the essence of what we call the scientific method. In such a scenario we do not need science centres or museums – theme parks and shopping malls will suffice.

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# CG12-15: Thinking about new museums

By M. Alsina (editor)

«It's easy! Well, it's not so hard! I mean: it's hard, but is fun!» «Big or Great? Size doesn't matter»

Conversation groups 12 and 15 were related to think about new museums of mathematics. They dealt from different points of view, but in order to give a more global view, it makes sense to join here some opinions and contributions on that.

By one side, in GC12, the questions were:

#### Is it really so difficult and expensive to build a new mathematics museum? Which elements are essential? How to stimulate demand in the area?

The discussion was leaded by Fernando Blasco, who explained the idea to create a museum of mathematics in Madrid in order to develop there more outreach and educational activities. See his paper, following this one in this proceedings, for more details.

Alfonso Peres Osia also presented a new incoming educative project by the Museo Nacional de Ciencia y Tecnología (MUNCYT), located at Alcobendas (Madrid) related to an Escape Room developed around mathematics.

By the other side, the discussion leaded by Tim Nissen in GC15 dealt with other features in the proposals for a math museum. Questions to promote discussion were:

### Does it make sense to discuss the ideal dimensions of a mathematics museum? Are we able to decide or suggest the best model or different models are equally possible and effective? Which elements are essential for a good activity?

Some answers can be found at the contribution by Guillermo Fernández in this proceedings.

# A museum of mathematics in Madrid

By Fernando Blasco

From some years ago a group of mathematics teachers want to set a Museum of mathematics or, at least, a space where maths exhibitions and activities can be done in a regular basis. The collection of 2000 educational mathematical pieces of Sociedad Madrileña de Profesores de Matemáticas Emma Castelnuovo have been restored recently by volunteers and the main purpose is to show them to people. On the other hand, activities such as "Maths in the Street" have lead to some members of the Society to develop new table and hands-on activities that can be included, sometimes as exhibition materials and sometimes as workshops, in a future Museum of Mathematics in Madrid.

As we know the experience of our colleagues in the Museum of Mathematics of Catalonia (MMACA), we think it is a good strategy beginning with temporary exhibitions and puntual actions to show both authorities and citizens the importance of mathematical education with manipulative materials. In many of the cases people that are responsible for giving grants and permissions for activities are not aware of the potential people that enjoy mathematical faires, and we have to show them that those activities are interesting for the general public and they are eager to participate in them. Mathematics, and Science in general, should be regarded as a part of culture by the social and political authorities to enter into people minds.

Our experience has begun organizing a "Maths in the street" activity in Madrid. Once we offered the idea to the local authorities they agreed to develop the activity once, but they did not trust in it: they did not think there would be crowds of people eager to participate in maths activities. For the second edition authorities already knew the power of maths on offering activities to general public and they gave us more space and more materials to organize the activity. The same with the third edition: the number of activities exposed has been growing and, even, authorities want to do it in other parts of the city.

Now, in this course, we will begin offering activities and exhibitions in a civic center as a new attempt to show the importance of mathematics and the expectation it creates in people. We pay attention to children and teenagers because they will live in a world where mathematics is a key part of technology. We do not think on mathematics just as a tool but as an area of knowledge that has its own history (and should be known), some educational materials (that can be shown) and a lot of ideas to stimulate creativity in the 21st century.

# The museum of contemporary science and social transformation

By Guillermo Fernández

Only in the last twenty-five years has the number of science museums in the world multiplied tenfold. This rapid and exciting progression brings with it important challenges.

The function and social role of museums have made rivers of ink, particularly in recent years, during which there has been a broad debate on this issue; a debate that has led to even some confusion in the sector. The debate about the function of the museum of contemporary science, actually has the same basis as an eventual discussion about what any language is for: naturally all language is a means that will serve those purposes that those who use that language determine or intend . The museographic language, like the cinematographic language or like any other, can be used for very diverse purposes: in the same way that it is possible to film a box office movie for profit or to film a film committed to a specific cause dedicated to helping change certain behaviors social.

It is common for many entities in the social sector to present difficulties in the development of their strategic management, despite the fact that they enjoy very intensive executive management. It should be emphasized that science museums act de facto as non-profit organizations with a social purpose, and that is why they share many of the management characteristics of the entities of the third social sector. In any case, many science museums often show strategic difficulties in determining what their social function is, or in order to achieve a clear purpose (and not just a good intention) in which to lead the cultural leadership in their society that must be their own.

One of the reasons why it may be difficult to study the social function of science museums is that they sometimes proclaim a series of social purposes that, paradoxically, often do not coincide with what they subsequently work on, or what they evaluate as success indicator. Thus, it is surprising that some museums declare that they intend to promote scientific opinion among the population (for example), although at the moment of truth what they evaluate mainly as an indicator of success is the number of visitors they obtain, but not the levels of opinion. scientifically encouraged, as would seem logical. It is really difficult to evaluate a qualitative objective such as fostering scientific opinion using, above all, a quantitative tool such as visitor counting. In any case there is a very effective rule: to know what is the underlying reason why something is done is enough to observe what is evaluated.

Some museums or exhibition projects are in practice lucrative initiatives. To some extent

this could be an oxymoron, in the sense that the definition of the International Council of Museums (ICOM) is explicit in this case identifying the museum with a non-profit entity. However, the museographic language can be an excellent means - very legitimate - to obtain huge economic benefits, as the specific case of some itinerant exhibitions (commonly called blockbusters), dedicated to a series of subjects that are usually quite recurrent and popular interest assured, such as dinosaurs or the wreck of the Titanic. Naturally, in this case what is generally valued as the main indicator is the number of visitors who satisfy their entry ticket, although sometimes certain social and disclosure purposes are proclaimed. In this line, some zoos and aquariums created with an eminently lucrative purpose could also enter (we insist: always that we attend to what they value mainly as an indicator of success) that actually fully and successfully use the resources of the museographic language. We must bear in mind that zoos and aquariums can be considered as full-fledged museums, not only because they are explicitly accepted by the ICOM definition, but also in the sense that they exhibit a collection of tangible pieces of great singularity and attractiveness. So tangible are those pieces that are alive.

Other management styles of science museums are based on being a reference for tourism in a particular locality. What counts here (again always in terms of what the museum evaluates as an indicator of success) is usually the number of visitors, despite the fact that other intentions related to scientific dissemination are sometimes proclaimed, although at the same time of the truth is not evaluated its achievement. Sometimes, these types of museums run the risk of trying both, but it is usually very complicated to combine a management aimed at getting tourists with another aimed at making an impact on the population's scientific literacy, because to a certain extent both purposes require styles antagonistic of management. One, the first, based on getting short-term visits using means that could be labeled as industrial, and another, the second, in pursuit of long-term educational impact based on a management that can be said based on artisanal. The result can end up being a museum that does not fulfill any of the purposes: as a tourist product you can find proposals capable of attracting more people, and as an informative project you can not achieve transformations with transient meaning in tourists.

A science museum can also be managed as a leisure or leisure establishment to which a cultural approach has been applied. To a great extent, what is offered is an entertainment style with a marked playful accent, as it is easily detached from the advertising messages of this type of centers. In these museums assets of intense aesthetic appeal that have certain manifestations or scientific phenomena are used to propose, de facto, a leisure alternative, usually very directed to family groups. It also evaluates and especially the number of visitors, and the scientific dissemination produced is usually like the value in the old mili, it is supposed.

The risk of this last style of museum management is that it can affect the concept and role of the science museum in the society of the 21st century, encouraging the possibility for citizens to assimilate the science museum to more leisure time equipment, and putting in some way the science museum to play in a league in which there are large establishments that are more effective than a museum of science if it is about entertaining. Thus, there is a risk that the science museum will thus abandon its fundamental and singular role in its society, to become diluted in the ocean of leisure consumption industries, where it can not compete at all and where it only runs the risk of ending up appearing before your community as redundant and dispensable equipment.

Transversally, many science museums have in their relationship with the school a basic pillar of their management, since they obtain a large part of their visitors. It is evident that at present there are many support needs of all kinds that the education system requires, but in any case it is essential that the science museum be related to the school always based on complementing its work, using the assets of the language that belongs to the museum.

## For a transforming science museum

But you could also opt for a science museum with a transcendent and relevant dimension: a very special and deeply social space that fully uses the resources of the museographic language to disseminate science in dialogue with other disciplines and to improve the intellectual resources of its community; a museum that truly seeks to change the people who visit it and not just tell them.

The transforming science museum is not obsessed by the number of visitors, because its management is based on a non-industrial and artisanal process that aims to achieve goals that go far beyond just entertaining, trying to put a lot of museum in people before many people in the museum. This type of museum accepts management mechanisms similar to those of other entities of the third social sector and enjoys the broad and long-term vision of a consolidated strategic direction that exercises true leadership over an executive action focused on efficiency. This type of museum shows that what it really wants is to make an impact on people's lives, stimulating them to search for knowledge, since that (and precisely that) is what they systematically evaluate.

The Transformational Science Museum is a museum of and for its local environment but at the same time has a universal vocation, which aspires to hold true cultural leadership in its community, and for that it encourages the participation of visitors and their complicity in all possible ways. And he thoroughly studies his social environment, although not to get more visitors, but to get more transformations. The transforming museum avoids the advent of dynasticism, reductionism or complacency, and is committed to assets such as knowledge, experience, high vision and the systematic search for excellence within the framework of a cohesive human team and as passionate as adequately formed.

The transformative museum ensures that it is offering something alternative and complementary to what other languages can offer, so that the museum conforms to a truly unique intellectual experience, as it is the basis of the exercise of quality museographic language, based on indigenous tangible assets of this language. The museum of transforming science investigates and develops the museographic language with enthusiasm but also with its feet on the ground, because it is not interested in megalomania but in the results. The transforming museum works with the school and not for the school and is a fully accessible and inclusive museum. And if it were necessary to look for an Anglicism for the social function of the transforming science museum, enlightenment would be better than entertainment.

The museum of transforming science is also a possible option and deserves, as soon as possible, its opportunity.

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# **CG16: Museum and University**

By Montserrat Alsina (STEAM projects coordinator EPSEM, Universitat Politècnica de Catalunya)

«How could I explain it to you?»

Nowadays science popularization has become an important social goal, developed by different agents and institutions. But, how do we disseminate advanced mathematic results? Is the museum the right place to do it? How is it related to universities?

Pere Pasqual, the moderator of this conversation group, introduced the topic and the questions to the participants, members of several universities and outreach associations. It is clear that universities are in charge to translate knowledge and information mainly to students, but also to society. But when talking about higher mathematics to a more general audience, sometimes people don't feel comfortable enough. The question we ask ourselves is *how could I explain it to you?* 

Participants exposed their opinions and experiences, and also shared their concerns and answers. Most universities publicly states in an explicit way as a mission to contribute to society. Therefore, participants coming from universities fully agreed that university community must be involved in the dissemination, especially in the research outreach. However it is not clear how to carry out this commitment and which relationship between universities and museums would be necessary. More collaboration in both senses would be good, and some effort need to be devoted to that.

Several particular features related to outreach of advanced mathematics were discussed. Next, some brief description of ideas is included.

Firstly, the relationship between mathematics and the real life was considered. Explaining how mathematics is useful in real life is a crucial point to make mathematics more interesting and valued by the general audience. Very often people is not aware of that, and reduce mathematics simply to numbers. However there is some objection, related to confuse it with the reason of being of mathematics or the need of justification. Mathematics is also like art, with a component of pleasure, aesthetics... Thus to justify ourselves must not be the main goal.

Another feature was to balance up to what level mathematical content need to be explained; in other words, to discuss if the audience needs to understand everything. Talking about research dissemination, this is a very important point. It is good that the audience can (immediately) understand something, as this make them to feel more comfortable and happier. But it is also good to show the difficulties and to say clearly when a simplification is made with the aim of facilitating understanding. Also, the value of mathematical rigor can not be despised, as it gives us security to everybody, experts or amateur. Actually,

mathematics can be introduced using some magic tricks, to arouse curiosity and questions, but the real magic is the power of mathematics as a solid and rigorous structure. Thus it is important to translate also that feeling to the people. A participant suggested music as a metaphor: people can enjoy a musical piece without knowing the score or playing any instrument. In this case it is not a magic result, but consequence of talent and effort.

Emphasis was also placed on communicative skills in order to guarantee a good quality of communication. Variety of resources can be used and some participants gave examples from their experience. Professionalisation of science outreach is interesting, but researchers can not be left out. It is good they do part of the dissemination by themselves, that allows they become closer to the society, helping to break up the image of being strange and unsociable. But to improve and to be successful training in this direction is really important.

As a conclusion, sharing resources and complementary points of view by university community and museums is essential to keep going on real outreach and dissemination.

Part III

# **Outreach Activities and Projects**

# 7deMATES project

By Montserrat Alsina et al\* (7deMATES working group)

The role of museums, schools and educational centres or other institutions is unequivocal and essential for the dissemination of mathematics, or science in general. But to be successful, initiatives and people to carry them out are essential. The project 7deMATES is an example of good practice in this direction. It was born as a particular programme of activities to offer interesting mathematics to young people outside the formal environment of school. But it has become more than an initiative, it is actually a great team to develop mathematics awareness mainly to teenagers and teachers.

Thus 7deMATES is a working group, consisting of high school and college math teachers, to foster knowledge of mathematics and STEAM. It works as a laboratory: for young people, for teachers and even for families. For young people, the objectives are to encourage the development of their skills, offering extracurricular and interdisciplinary content, in a natural environment with people of the same age, enhancing their protagonism and active participation. For teachers, it is a space to develop and innovate, working as a team, experimenting with new materials, sharing the limelight with young people and building bridges of collaboration between secondary school and university teachers. For the families, it is an space, an opportunity to share activities outside formal environments and to give to their children an environment where they feel challenged.

### **Origin and goals**

The program was born in 2015 in the region of Central Catalonia, called *Regió 7*, inspired by the real experience at a personal and professional level. It was detected that young people were thirsty for math, and that gave rise to the program and its name (in Catalan, thirst and number seven are written the same).

Having as target young people, the profile of the programme was defined taking into account the location, namely territoriality, and other programs, so it was aimed at students on 2nd-3rd year of secondary school, prior to the baccalaureate or other non-compulsory level. It is therefore defined as a complementary project, with main objectives: a) favoring the development of the capacities of young people who show potential for mathematics, without asking them to give up other extracurricular activities; b) offering extra curricular content for in-depth study of mathematics, which does not interfere with the usual development of the curriculum in the school context; c) encouraging boys and girls to take an active role and to work as a team.

The proposal consists of 7 sessions during the academic year, approximately in a monthly

basis (between October and May), preferably on Saturday mornings, so that boys/girls from other villages and cities in the region can participate, which favors territorial balance.

The programme also pursues contact with institutions, associations and companies in the territory, to establish complicity. STEAM also stands out as an added value, and professionals from other disciplines are invited to collaborate.

#### **Development and evolution**

The first edition was made public in September 2015, with several initial meetings encouraging teachers and students from the immediate surroundings to participate, with registration process directly through the families. The response of young people was spectacular, requiring the introduction of a selection test, since the number of participants had to be limited to ensure quality. The interest aroused by this mathematics laboratory initiative among the boys and girls of the region and their families confirmed the need and suitability of the project.

To run the project not only young people was needed but a team of enthusiast math outreach persons willing to devote time and energy to create and carry out activities and the suitable resources. Former experience in some outreach activities allowed to form a core group to get started, and workshops and resources for that year sessions were built.

In 2016, at the time to begin the second edition (2016-17), new highly motivated teachers joined the project giving stability and continuity to the working group. Thus currently, 2018-19 academic year, fourth edition is going on. Each edition the project attracts more people and the number of inscriptions, and the size of influence area in Catalonia, keep growing. Thus, it will be interesting to set other working groups in other areas.

### Methodology, content and attitudes

The development and methodology of a session is flexible and varied. It alternates expository and experimental part, induction and deduction, group and individual work, guided or freer development, cooperation and competition, etc. Thus young people and conductors can discover their potentialities in a creative environment. In each edition, the young participants are naturally renewed, and the accumulated experience affects the evolution of the project positively.

Topics of the sessions over the years have been very diverse, including among others: strategy games, graphs and topology, reflections on infinity, numbers from different points of view, secret sharing, polygons and tessellations, geometric maps, statistical experimentation and probability, ciphers, graphs, infinite sums, magic, etc. More ideas are always welcome, because the goal is not to be limited to a reduced and repetitive list.

Sessions are self-made, unique and adapted to the group. Thus resources and materials are

prepared for each session, according the topic, the experiment and the aim, so each session becomes a session-lab, an experimental museum for that day. Each session can be planned and enjoyed as a single-use session, what brings a sense of freedom and discovery. This is a significant aspect, which gives life to the working group, as members are not conditioned to act following the script, but puts their creativity into play. This builds a real team, and that is also transferred to the participants.

The editions carried out and the follow-up of the young people who have participated in the program allow us to affirm that it fulfills the motivation and initial objectives, focused on young people. However, as it was remarked, the experience has highlighted the value of the working group, because of the continuous team-building work and the preparation of materials, what counts as outreach training.

Currently, looking forward to the fifth edition, the project has consolidated itself as a real outreach laboratory.



(\*) 7deMATES working group components are (currently): Montserrat Alsina, Josep Ma Baron, Alba Blasco, Alba Carné, Glòria Casas, Josep Costa, Immaculada Gilibets, Gerard Gràcia, Noèlia Perez, Maria Angels Pinyot, Ester Plaza, Carla Rodriguez.

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# Nummolt Apps

By Maurici Carbó (Nummolt Apps)

We know that Exhibitions in Math museums are proven successful experiences. The materials, well explained by human assistants, involve the users. But that experience can be repeated at most a few times in a short timespan (in case of having a nearby museum) while smartphones are pervasive nowadays.

Since many years ago, there are inspiring interactive math webpages and applets dedicated to mathematics, like Illuminations, that give an attractive insight into the topic. There are also many interactive applets and simulations made with Geogebra. All of them were designed to be used with computers, workstations or laptops at school or at home.

In the museums of mathematics, unlike traditional museums, touching the exhibition is mandatory. The capabilities of smartphones are closer to that reality than the classic interaction that can be obtained with a keyboard or a mouse: We can interact by touching the screen with one or more fingers, selecting, dragging or pinching objects.

There is also the possibility to make analogies and animations that have no relation to the formal presentation to which we are used in the classroom.

Educational apps for sale are geared towards helping with homework: computing, word processing, evaluable exercises, etc. I am interested in virtual exhibits that stimulate the same desire to experiment, research and go deeper as the hands-on activities of an exhibition.

I would like to share here the experience as an amateur developer trying to take advantage of the tactile capabilities of Android smartphones and tablets for mathematics exploration, adapting some materials from my old works for www.mathcats.com/explore and creating new ones.

I've tried to make original math apps. Looking for things that I would have liked to see in my educational process. Some apps are elementary, and others are proofs of concepts that try to show what can be done. And many of them have been coded in collaboration with people from all over the world.

In December 2017 Aleix Mestre wrote about Delaunay Triangles and their corresponding Voronoi polygons. His work included basic concepts like the conditions for valid Delaunay triangles and the construction of the Voronoi Polygons. My reaction after Aleix Mestre's work was:

• **Voronoi Soccer** A soccer self playing interactive simulation where the players interact with their Voronoi polygons: www.youtube.com/watch?v=NahQ09R59zg

Here there's a small sample of my journey exploring the development of interactive and open-ended math apps:

• **Euler and Hamilton Path** Shortest Hamilton Path search (verified by brute force) and more. My first and only attempt to make a version of Math Museum materials.

www.youtube.com/watch?v=AVCjOIRCky4

• **Touch Integers** Addition, subtraction, abacus, multiplication, integer division and prime factors. Example of exploration: building Mersenne primes  $(2^p - 1)$ .

#### www.youtube.com/watch?v=sOnkpDihIS4

• Math Garden An analogy about Integers and agriculture. Integers as plants and seeds. Numbers as seeds for addition or plants for multiplication. To play with integers and prime numbers. Fractions are double seeds for addition and plants with flowers for multiplication.

www.youtube.com/watch?v=hCd\_xEIcW8k

• **Reading Fractions** Reading Proper and improper fractions. Inspired in M. Montessori's drawings.

www.youtube.com/watch?v=UavGd7zmlvo

• Fractions Balance An extension of Reading Fractions to explore division of fractions.

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# Math festival in Chile

By Mariela Carvacho (Sociedad Matemática de Chile)

In Chile from 2016 the Math Society of Chile began organized the Math festival. I was organizer and contributed with the creation of some of its interactive exhibitions.



Our main goal: How you may show the people that Mathematics is not just formulas and complicated computes?

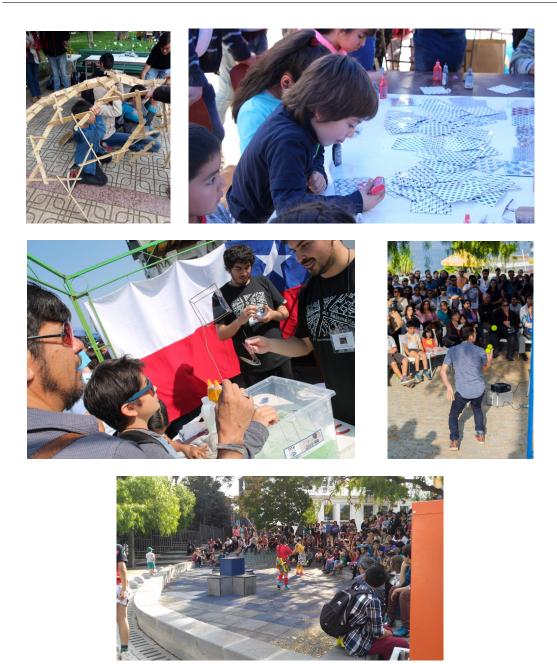
For us (how team of Math festival) think activities like Math festival are activities where people can explore, play and feel the Mathematics. My personal experience (phd studies) gave me a new perspective of Mathematics. The abstraction and working with deep ideas it allow me go to the origin of knowledge. It gives me ideas to show in an natural way different concepts of Mathematics. Finally we try to increase the curiosity and change the idea that Mathematics is a difficult science.

The Math Festival contains around 20 interactive exhibitions, theater, juggling show and open dissemination talks. Furthermore in each version we invited the public to participate in a concourse: take a picture or write a poetry or create an exhibition, with Math sense. The web page is

#### festivaldematematica.cl

Valparaíso (December-2016), Vicuña (July-2017), San Antonio (September-2017), Talca (December-2017), Valdivia (April-2018), Chiloé (October-2018) are the cities where the Math Festival arrived.

In general the people is very enthusiastic with this activities. One of the consequences is the interest of Math teachers to replicate some of the exhibitions in their schools.



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[1] festivaldematematica.cl

# Painting aboriginal art of Chile

By Mariela Carvacho (Sociedad Matemática de Chile)

Is Art related with Math?



Diaguita was a northern culture of Chile. Your vessels was decorated has geometric figures with certain patrons. With this activity we explored the geometric nature of Diaguita vessels from a mathematical point of view. These type research is well know for Islamic culture. In [2] Grünbaum gave a discussion about the symmetry groups present in the Alhambra.

Our idea is continue the studies given by Washburn and Crowe in [4]. This is a progress work in collaboration with P.González and A.Navas. According to P.González in [1] around 50% of Diaguita-Inca desings express the idea of four-partition. That follows by the spatial organization of Inca empire. On this set P.González distinguish the generated by the *doble reflexión especular*: a graphic desing of yanantin. Yanantin is the concept developed by Platt in [3] which associate oposite elements like women - men, right - left. P.González remark this is similar to Tawantinsuyu (it meas four places called suyus).

To identify the rigid movements is not an easy work. It necessary to refine your eyes. It for this reason that we invite the people to paint one of these design, vitral paint, and then use the patron with the glass to identify the different rigid movement. For the most curios persons we explain all paints are classify in one of the 17th wallpapers groups.

It is possible to see the idea in the following video:

youtu.be/Sm9ivP9ia1s?t=81

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# Mathematics Festival: A Broad Dissemination Project

By Carmen Rosa Giraldo (Federal University of Minas Gerais, Brazil)

Because of its duration and intensity, school experience dominates all other possible factors that may influence people in defining their attitude towards mathematics. It is therefore necessary to create opportunities inside and outside the school environment to show the usefulness of the mathematics taught, as it appears in different contexts of our daily life, including in the arts and music. Not less it is important to give the opportunity to experience less formal aspects of mathematics through tricks, games and puzzles, for example. Through these actions it is possible to change the conception of mathematics in the popular imagination.

The actions of popularizing mathematics have a long history. In the biography of David Hilbert [2], for example, the author describes disclosure conferences, given in 1921 by this prominent mathematician to students on their return to university after the war, that continued throughout this decade. At present, the reflection on actions of popularization of mathematics gains an increasing space between the community of professors and professional mathematicians. In the site www.mathunion.org/icm, for example, it is possible to consult different published works on this one topic in the summaries of the International Mathematical Congresses, organized by the International Union of Mathematicians (IMU). The International Commission on Mathematical Instruction (ICMI) devoted the 16th ICMI Study Conferences [1] to topics on the popularization of mathematics.

The project "Mathematics everywhere: activities for mathematics festivals in the state of Minas Gerais" was an initiative carried out by a group of professors from the Department of Mathematics of the Universidade Federal de Minas Gerais (UFMG) in Brazil, wiht the objetive to contribute to the popularization of mathematical knowledge and the improvement of teaching and learning of Mathematics through the realization of Mathematical Festivals. It was partially supported by CNPq in response to the Edict MCTIC/CNPq02/2017 in support of activities of dissemination and popularization of mathematics, during the National Science and Technology Week in 2017.

In its history, the Department of Mathematics of UFMG has worked with mathematics teachers and students of Middle ang High Schools in Minas Gerais, through various extension projects. These projects have always had a common purpose, which justifies their existence: to disseminate Mathematics, making it accessible to a wider public, and to contribute to the improvement of Mathematics teaching and learning in both Middle School and High School



Figure 3: Mathematics Festivals in Belo Horizonte and Iapú Minas Gerais

giving priority to public schools and promoting a closer articulation between these Schools and the University. This is part of the work plan of this department and its actions are included in the Extension Program for Teachers and Students of the Basic School, created in 2005 and registered in the Extension Information System of UFMG in 2010.

The Mathematics Festivals, held within the framework of this project, involved a variety of activities such as mathematical games, workshops, cinema, storytelling, exhibitions and speeches of dissemination, always seeking to foster a positive attitude towards mathematics and promoting a closer articulation between students and teachers of the Middle and High School and the University. The different actions offered to the general public, and in particular to students of the Elementary School, the opportunity to experience math in a playful way. In addition, activities aimed at adults were focused on divulging of mathematics and its applicability in daily life and interesting facts of its history and its relationship with other areas such as art and music.

The success of the first edition of the festival, held in October 2017 in Belo Horizonte, was followed by the great participation in the festival held on March 1 and 2, 2018 at UFMG (about 3 thousand students from 75 middle and high schools participated in this festival) and other math festivals in 2018 in various cities of Minas Gerais. The participants stayed at the festivals for about two and a half hours and were able to think and discover the mathematics by performing activities such as: observation of the nightsky, mathemagic, movie session, board games, login mazes, plane tilings, construction of kaleidoscycles, origami, flutes, tensegrity, etc.

#### Proceeding of the International III MATRIX Conference Outreach Activities and Projects



Figure 4: Mathematics Festivals for teachers

There was also, the participation of about three hundred teachers from public and private schools at a festival dedicated to primary and secondary school teachers, who sought to acquire knowledge, methods and references that allowed them to organize this type of activity in their classes. This was a good indicator of the fulfillment of one of the objectives of the project: to increase the articulation between students and teachers of the Elementary School with professors and students of the UFMG Course of Mathematics.

It is noteworthy that other projects of the Department of Mathematics of UFMG, such as the "Projeto Visitas" and "PET", contributed with the offering of workshops and ludic activities in the various editions of the festivals, thus extending its reach.

In addition, due to the development of this project the "Museum of Mathematics of UFMG" was inaugurated, currently located in Room 4011 of the ICEx-UFMG, with the goal of promoting mathematics through ludic activities, puzzles, strategy games, presentation of formulas and theorems with concrete objects, exhibition on mathematics and art, and logical challenges that encourage the interest of visitors, especially teachers, leading them to a reflection on proposals of compelling activities that pass a positive vision of the teaching and learning of mathematics.

Finally, throughout the development of the project we were able to:

- Contribute to the dissemination and popularization of Mathematics.
- Promote a close articulation between the students and teachers of mathematics of basic education with teachers and students of UFMG Mathematics Course and other teacher-training institutions, bringing the University closer to the public system of Basic Education and the population in general.



Figure 5: Museum of Mathematics of UFMG

- Present mathematics as a dynamic and accessible content, instigating the search for new knowledge, thus contributing to motivate teachers and students of the Elementary School to study Mathematics and encouraging them to the continued formation of Basic Mathematics teachers, enabling access to new educational techniques and practices.
- Provide the participation of students of the Mathematics course in research, teaching and extension activities enriching their academic training of those who will participate in the project.
- Introduce to the public the subjects of recreational mathematics that contribute to arouse the interest in mathematics and to improve the acceptance of this discipline among the students.
- Ensure that the activities developed by the extension and undergraduate projects of the Department of Mathematics of UFMG can benefit a wider public.

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# DIVERMAZO: The Mathematicians deck of cards

By Nelo A. Maestre (DIVERMATES)

We have created a deck of cards with the 72 most important mathematicians of all time.

"It might surprise many people the closeness between the mathematician line of thought and a playful one. It is a well known fact that a great deal of the most serious math discoveries have been developed from an initial ludic point of view. It is well known that puzzles and mind games share with mathematics many mental processes that are efficient in both fields. Furthermore, in many ocassions it is surprisingly hard to decide when the game ends and the "serious" scientific activity begins. To many mathematicians math will never stop being a game"

"Para Pensar Mejor" by Miguel de Guzmán 1994

Inspired by the words of Miguel de Guzmán, we created a deck thinking in the idea of a versatil set of cards design to play many games, some of them versions of commercial or classic games, some others created by Divermates. All these games are thought to attack a distinct part of the brain. You will play arithmetic games, spatial vision games, games based in inductive or deductive logic, you can play no games, games with winner strategy, cooperative games, games with or without an interturn. All these games are design to be played depending on the environment where you want to play them.

Each card represents the face of a famous mathematician. It has the name, picture and date of birth and death of the mathematician as well as two different indexes. These two indexes and the colour of the card will be used to play the games. Organizing the cards by the black index you will find yourself with a timeline of the history of maths.

Downloading the HP Reveal application you can find one more interesting use of the cards. Placing the card in front of your cell phone will make appear in augmented reality a brief text with the history and discoveries of the mathematician.

Divermazo is a tool specially design to be used in all kind of academic environments, as well as homes, and while playing games and having fun you will be learning the names and history of the most important men and women in maths history.

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[1] divermazo.com

# Going further than a moment of fun with math

By Aniura Milanés Barrientos (Federal University of Minas Gerais, Brazil)

# **Our project**

At the Mathematics Department of the Federal University of Minas Gerais (UFMG), in Brazil, we have a Maths outreach project that I coordinate where a team of 8 Maths students interact with groups of elementary students who visit us because of our fun Maths activities and games.

The visits take place from Tuesday to Friday from 09:00 to 11:00 and also from 15:00 to 17:00. The age of the visitors ranges from 10 to 17 years. We also receive older people that are enrolled in the Education Program for Young and Adult People and visit us in the evening. In general, two teachers accompany the students on the visits, but they do not participate in the activities.

In each visit, there are many different types of activities. There could be games, competitions, puzzles or magic tricks and all of them have a mathematical background.

# A sample activity

There is one activity that is a kind of symbol for us and that gave rise to the idea on which our logo was based. It is called The game of sides.

- 1. Five equal disks are needed, with faces of two different colors (for example, red and yellow).
- 2. The visitors choose some initial configuration for the five disks.
- 3. Then, one of the tutors observes the chosen configuration and abandons the room.
- 4. The tutors that remain ask the visitors to turn over a disk of their preference five times.
- 5. Then they ask them to choose one disk, to memorize the upper color and to hide it.
- 6. Finally, the other tutor enters and discovers the upper color of the hidden disk.

## How does it work?

After the five disks are flipped, the parity of the number of red and yellow upper sides changes with respect to that in the initial configuration. The "guessing" tutor remembers the parity of each color, then she or he is able to know the parity of each color in the final configuration and therefore discover the absent color.

For example, if the initial configuration has four red disks and one yellow, we would have an even number of red upper sides and an odd number of yellow upper sides. After the five turnings, we would have an odd number of red upper sides and an even number of yellow upper sides. So, if among the four visible sides we see an odd number of red upper sides, then the hidden color is necessarily yellow. Otherwise it's necessarily red.

This is a "guessing" activity that astonishes the visitors. Sometimes they even ask to accompany the guessing tutor outside the room to check that nothing "illegal" is happening. This activity was designed by several students and is based on the trick Heads or Tails? in [1].

## Our proposal

It is always a big challenge to hold the interest of our guests until we can discuss with them all the details about the maths behind the amusing activities they just experienced. It can be particularly difficult with some activities with specific groups.

In either case, since the duration of the visit is only two hours and since the students will hardly ever come back to visit us, it is essential to develop concrete actions that promote a deep understanding of the ideas behind the games and tricks and the pleasure of engaging with maths. To achieve this goal, we propose the following ideas.

- 1. To continue organizing workshops for teachers' training.
- 2. To create some materials where we put together different engaging math activities organized in accordance to the ideas that they carry in the background.
- 3. To complement these materials with lesson plans that bring to light and discuss the mathematical ideas involved.

We would like to remark that this project has been built over many years by many enthusiastic professors and students engaged with the task of bringing mathematical knowledge to the public.

### References

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# Atractor

By Ana Cristina Oliveira (Atractor)

Atractor is a non-profit association created in Portugal for the popularization of Mathematics. Atractor gained a large experience in producing virtual contents, offering a wide range of resources. Several Atractor's resources are summarized in the following lines. Some more details are given in her Conversation group contribution.

Atractor's website (www.atractor.pt) contains more than 2.000 web pages devoted to the popularization of Mathematics, illustrated by a large number of animations, images and interactive materials.

Some *freeware software* developed by Atractor (see www.atractor.pt/soft.html) is:

- **GeCla** a program which allows creating friezes and wallpaper with a given symmetry or classifying existing ones)
- **AtrMini** a set of virtual games aimed at children, both for computers and tablets/smartphones)

Also there is a large project, under development, for the study from the mathematical point of view of the patterns found in the Azulejos of the houses all over different cities of Portugal.

Moreover there is the *Atractor's YouTube Channel* with 20 small mathematical films, entirely produced by Atractor: at www.youtube.com/channel/UCymZYeiV6b-86ZpDM3\_HspQ.

# A touring mathematics laboratory: The Magic Mathworks Travelling Circus

By Paul Stephenson (The Magic Mathworks Travelling Circus)

## Introduction

As The Magic Mathworks Travelling Circus nears its 30th birthday, these Proceedings provide an opportunity to review our practice over this period in the hope that aspects may be relevant to readers starting out 'on the road'. One thing we can say at once, which will come as no surprise: what works for children, works for adults; and what works for children in one country works for children in another.

We are constituted as an incorporated charity, registered in the U.K. This constitution has little relevance for international readers. For them it is enough to know that it provides a certain stability and continuity, important when seeking academic grants, and it allows us to employ people, though for the most part we contract services. Students or young graduates employed on the latter basis we call 'mathworkers'. We pay them at an hourly rate but the chief benefit for them is the experience. They may add this to their C.V.s and we are happy to provide testimonials for prospective employers.

### Goal

Our goal is to help teachers of children of school age. What we provide is a resource, no more. Though, as you will read, our methodology is very specific, we are not advocating a pedagogic method. Indeed the nature of individuals and their brains is such that there can be no unique means by which an understanding of mathematics may be acquired.

# Strategy

We take our exhibitions to many kinds of venue, serving many different visitor groups and types of event but we can best define our strategy by describing a typical event. We set up our touring maths lab in a school, where we aim to provide a multisensory environment in which mathematics can be experienced. For the teachers it is a laboratory of the second order: they can step back and observe what particular children are responding to, in order to use those particular materials in their day-to-day teaching. We hope that the children enjoy the experience and the affective dimension is particularly critical where mathematics is concerned. But we are not expecting the children to master a topic in the course of their experiments. Our hope, nonetheless, is that they will have taken ownership in the psychological sense of at least one station, which has given them an insight they can celebrate. However, in schools where we are resident for a week or more, there is the opportunity for them to choose a particular station in consultation with their teacher and make a particular study of it. The result may be a written report or a presentation given to classmates or others. It is also the case that we revisit schools on a regular basis (typically every 3 years) so the teachers are already familiar with our materials and therefore in a better position to help the children. We maintain a website for the benefit of both teachers and students, who will find there The Virtual Circus, containing sets of interactivities through which the activities in the real Circus can be extended.

Accordingly, here are the recommendations we make to intending hosts: Though a Magic Mathworks visit forms a brief part of the pupil's year, it is important that he sees it as part of his mathematics timetable. If the visit extends over a week, the pupil should ideally make two visits, separated by a class lesson where the teacher can prepare him for the return. Within a session, intervention should be the minimum necessary to ensure the children can enjoy the activities. This should leave the teacher and other supervisors free to observe the pupils [1]. Many of the activities can be amplified in physical scale and done as 'people maths'. An obvious example is the 'Handshakes' problem, where groups of children can shake hands. All the activities depending on a grid or strip of squares can be done by children sitting in a block of chairs, colour-coded by a hat, scarf or sweatshirt [2]. In a schoolbased maths lab there should ideally be the greatest possible range of materials available: stationery, manipulatives, models, and there should be access to computer stations where mathematical software can be run and programs written and executed. In our touring lab we generally confine ourselves to physical materials and leave for preparation and follow-up what can be accessed via our website. Only if time in a second session allows do we give the children the opportunity to use a virtual extension of a real activity [3].

# Methodology

Here are the five principles on which our exhibitions are constructed and the research base justifying them.

#### A multisensory environment

The brain accepts input by any and every sensory pathway. If we acquire non-contradictory information by two sense modalities, the effect of the combined input exceeds that from either source alone.

We begin with an everyday observation. Babies grasp objects, shake them, and put them in their mouths: if we use a number of senses to obtain information at that age, there is no reason to suppose that we should not do so at a later age. Schools inspired by the teaching and writing of Maria Montessori make this assumption. There are many cases where only by combining stimuli do we resolve a question: we see Anne's lips move and know that it was Anne and not Jane who spoke. There are survival advantages in sensory redundancy: fleeing a fire, we see flames, hear a crackling sound and smell smoke. We only have to cast our minds back to our earliest memories to know that vivid memories are multisensory: what comes back to you is not just the appearance of a desk lid closing but the sound, and the smell of wood varnish. You wish to identify song birds by sight alone. The evidence from other experiments is that you will do better to hear the song of each bird while studying its picture than merely to study its picture. It used to be thought that the brain processed inputs from the different senses individually before combining them. But it now appears that combination occurs from the earliest processing stages [4].

#### **Multiple embodiments**

Multiple embodiment was enunciated as a principle by Z. P. Dienes [5]. Dienes' idea was that the child who abstracts the feature common to the disparate instances can be said to have understood the concept in question [6]. There is only one way the brain can do this and that is by perceiving the analogy between them. Keith Holyoak and colleagues have identified the brain regions active in explicitly analogical tasks. Holyoak's team developed a computer model called LISA (Learning and Inference with Schemas and Analogies). How, fed minimal constraints, it infers relations between concepts, they find to be comparable to how the brain does so, inferred from brain imaging studies [7]. In our exhibitions we use signage to make the concepts explicit and the grouping is there primarily for the teachers so that they can see how a variety of apparatus and found objects can be used to serve a mathematical notion.

#### Collaboration

When we collaborate on a task, we contribute to the group our understanding of it and, by teaching, we learn – in Seneca's Latin phrase, docendo discimus. Recent experiments with very young children have shown that to teach, to help another learner, is instinctual [8, 9, 10]. An important observation is that body language and gesture enable young children to convey what they cannot with words. If you take this non-verbal communication into account, Piaget's claim that, for example, young children do not conserve number, is undermined. No discussion of collaboration should omit reference to Lev Vygotsky. For students in the west an edited collection of Vygotsky's papers (spanning the period 1930 to 1960) was issued as 'Mind in Society'. Vygotsky wanted to supply what he saw as a deficiency in Piaget's cognitive psychology: the learner isolated from the community. From an evolutionary viewpoint he saw the brain and its activity as a microcosm of the tribe:

"All the higher functions originate as actual relationships between individuals" [11]. The artificial intelligence pioneer Marvin Minsky had his own, distinctive take on the idea. The mind, which is no more than the activity of the brain, is the net effect of the activity of independent agents (subsystems of the brain). These agents people the mental society. His model has stood up well. Thirty years on, many of these agents have been identified anatomically and physiologically. When we experience a thought, the correlate in terms of brain activity is that, at that moment, a particular agent has won control of the mechanism of attention [12]. Vygotsky's insight suggests that we can also move back up in scale, from the individual brain to the community of learners. This happens of course in the dissemination of research. But it can also happen within the classroom. In 1987 Alan H. Schoenfeld published the edited proceedings of a conference aimed to bring cognitive science into the maths classroom, just as John Geake attempted to do for the whole school in 2009 [13, 14]. As a footnote to his chapter on metacognition, at the point where he's discussing the dynamics of small groups of students engaged in solving problems, he writes:

I have thought about slightly less natural small group interactions, designed to promote particular characters in the society of mind [he takes Minsky's phrase]. Suppose you structured small groups so that individuals had one or more assigned roles – say as idea generator, critic, monitor, etc. ... This was exactly what Jo Boaler went on to do [15].

### A low threshold but a high ceiling

Like any other public exhibition, The Magic Mathworks may receive at any given time visitors with a wide range of knowledge, experience and ability. Even in a school classroom, the range will be great. It therefore goes without saying that the threshold should be low. Even so, there should be some dimension of challenge. For the same reason, the ceiling should be high. The key finding here is that the brain craves a degree of difficulty which is sufficient to motivate but not so great that the learner is demoralised. The Bjorks coin the phrase desirable difficulty to describe this [16]. If the child is 'stretched', to use a common term - or, more importantly, stretches himself - the concepts with which he struggles will be more securely lodged in long-term memory.

### Enjoyment

We say "Nothing succeeds like success". What the saying illustrates in terms of the brain is a positive feedback loop. The reward centres in the brain are activated and they in turn incite further activity of the same kind. This need not be pleasurable in an obvious way. A musician who plays a scale well will be encouraged to practise more. The performance becomes a reward in itself. The factor is obviously critical in mathematics where, to fail to solve a problem, can too readily be experienced as failure full stop [17].

## **Evaluation**

Because our activities are, at the worst, non-threatening, and, at the best, enjoyable, most children, and therefore most teachers, leave our exhibitions happy. Given sufficient planning and resources we could over our 30 years have conducted a major longitudinal study and attempted to analyse the long-term benefits of the experience. This we have failed to do. Statistically it is of course difficult to decide what to measure. The children's experience with us, though we hope inspirational and motivating, is fleeting. How does attending a symphony concert improve one's appreciation of classical music? It would be hard to claim that it did not, but equally hard to determine precisely how the improvement was achieved. Nevertheless, any organisation embarking on an outreach programme should have some system of evaluation in place tied closely to the goal enshrined in their mission statement. One thing we would like to have determined is the link the children perceive between their day-to-day mathematics and what we provide. Our topics are standard in the curriculum -'Multiplication', 'Symmetry', and so on - so that much is clear. But, though the staff of the schools we revisit regularly and for long periods are at pains to make the link, we may be in a school for just one day at the end of a summer term. In this case the children may simply see the event as an add-on.

## References

- [1] From the www.magicmathworks.org menu, choose 'Preparing for a visit'.
- [2] From the www.magicmathworks.org menu, choose 'Masterclasses', then 'People maths'.
- [3] From the www.magicmathworks.org menu, choose 'Virtual Circus'.
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- [11] M. Cole, V. John-Steiner, S. Scribner, E. Souberman (eds.), 'Mind in Society' (1980).
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[16] You-Tube video 'robert bjork – desirable difficulties: slowing down learning', bjorklab.psych.ucla.edu.

[17] Wikipedia entry 'Mesolimbic pathway'.

