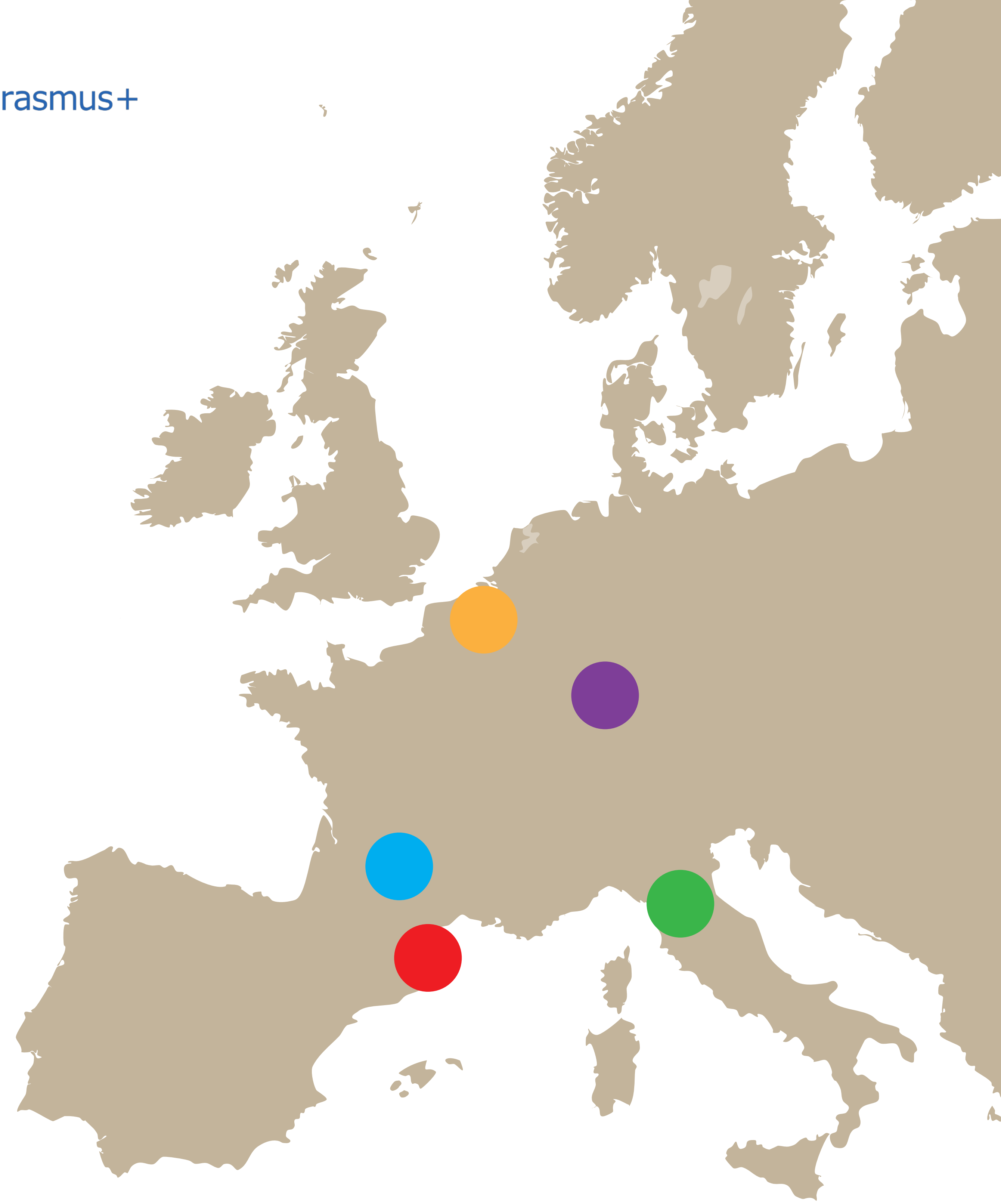
Erasmus+



A non-formal approach to mathematical education

mathspaces

Intelectual Output 01 for the project by MdM-ERASMUS+.

This project has been funded with support from the European Commission.

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MMACA is the coordinator of this booklet, responsible for its design, revisions of the text and the analysis of the graphics.

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0 The Mathspaces project

Mathematics is a cornerstone in education, and it is fundamental to understand many challenges of modern society. However, in the public perception mathematics is still one of the topics more attached to school and formal education. Some associations and outreach projects try and work to approach mathematics to students and also the general public in out-of-the-school environments, such as public exhibitions, workshops, fairs, museums, etc. that one can call Math Spaces. The five partners of the Mathspaces Erasmus+ project, La Maison des Maths, Fermat Science, Il Giardino di Archimede, IMAGINARY, and MMACA; have a proven experience on organizing and running such math spaces, and a solid background on education and science communication. This booklet aims to share the views and experiences of the Mathspaces partners on the impact of such activities on the learning process of children (and also adults), using this non-formal approach to mathematical education.

In a revealing study on experimental sciences learning, J.H. Falk and L.D. Dierking [1] report:

[...] Data from the Programme for International Student Assessment showed that a major predictor of high achievement on the test was participation in out-of-school, free-choice learning experiences such as visits to science museums.

[...] None has stated it so clearly and forcefully as the Harvard Family Research Project [2], which stated: The dominant assumption behind much current educational policy and practice is that school is the only place where and when children learn. This assumption is wrong. Forty years of steadily accumulating research shows that out-of-school or "complementary learning" opportunities are major predictors of children's development, learning and educational achievement. The research also indicates that economically and otherwise disadvantaged children are less likely than their more-advantaged peers to have access to these opportunities. This inequity substantially undermines their learning and chances for school success.

The PISA project of the OCDE (1997) [3], the Framework of Key Competences for Lifelong Learning (2006) [4] and its review (2017) [5] represent important opportunities to open a dialogue between formal and non-formal educational institutions to adapt the learning models to new times and challenges.

1 J. H. Falk and L. D. Dierking (2010). "The 95 Percent Solution". American Scientist, volume 98: 486-493

2 Harvard Family Research Project (2009) http//www.hfrp.org/content/download/1072/48575/file/findings_predictor_OSTfactsheet.pdf

3 http://www.oecd.org/pisa/

4 http://ec.europa.eu/education/policy/school/competences_en

5 https://ec.europa.eu/education/initiatives/key-competences-framework-review-2017_en

PISA results (2012) showed that 22.1% of European students were low achievers in maths. Which means that more than one out of five youngsters in Europe is not equipped with the basic skills necessary for numerous valuable jobs in our current economy.

The Commission has the objective to lower this number to 15% in 2020 but, so far, in many countries the progression is weak when the level is not simply lowering.

One of the reasons for that is the lack of engagement of youngsters who have a very negative perception of mathematics: most kids start mathematics with the implemented idea that mathematics is difficult. To counter that, a movement has been emerging in the last 10 - 15 years all around Europe with the creation of museums, centres or houses of mathematics to promote a non-formal approach to mathematics which has a proven effect on the mathematical skill of youngsters and, above all, on their engagement with mathematics.

However, the spaces dedicated to this approach to maths are still only a few... This shortage is due to the lack of knowledge about the approach and the difficulty to find the appropriate resources and content. So the partners, with the clear objective of addressing underachievement in the basic skills of maths, gathered to create a project that aims to increase the awareness and practice of these spaces dedicated to a non-formal approach to mathematics in the European Union.

So, our project collects the suggestions that Falk and Dierking propose at the end of their article, even if they are related to the STEM learning and not just of mathematics and to the US reality, which is not exactly the case of European countries:

[...] We are not advocating any diminishment in the efforts to improve and expand school-based science education, [...] We would argue for increased efforts to measure the cumulative and complementary influences of both in- and out-of-school science learning. However, given that at present school-based science education efforts receive an order of magnitude more resources than free-choice learning options, even a modest change in this ratio could make a huge difference. The data suggest it would be a wise investment.

mathspaces

ASBL Entr'Aide,
La Maison des Maths

Typology:

Non-profit association

Office:	Public:
Rue Jules Destree, 121 7390 Quaregnon (Belgium)	Rue du Coron, 105 7390 Quaregnon (Belgium)

Web page:

http://maisondesmaths.be

Contacts:

maude@maisondesmaths.be
https://www.facebook.com/maisondesmaths
https://twitter.com/MaisonDesMaths

Principal activities:

- "Come and discover a mathe-magical world"
- Hosting of school groups every day of the week (nursery, primary, secondary, high schools, universities …)
- Teacher training on non-formal math (from nursery to higher education)
- Organization of "mathiversaries"
- Organization of traineeship "Mathins Malins"
- 3 open-door days per year
- A shop that sells rigorously selected games for their mathematical rating
- Local and regional fairs
- Showference: "The Very Math Trip"

Principal projects:

- Erasmus+ project : MathSpaces
- Erasmus+ project : Informath
- Opening of Mathalis, a new Maison des Maths, in Liège, January 2018
- The MathLab : 1st European movable laboratory to experiment mathematics

Principal supporters:

- Google
- Mathematikum
- La Région Wallonne
- King Baudouin's Foundation
- Fédération Wallonie-Bruxelles
- Erasmus +

FERMAT SCIENCE

Une autre idée des maths

Fermat Science

Typology:

Non-profit association

Home:

Maison de Fermat
3, rue Fermat
82500 Beaumont de Lomagne (France)

Web page:

https://www.fermat-science.com

Contacts:

thomas.fermatscience@gmail.com
contact@fermat-science.com
https://www.facebook.com/FermatScience
https://twitter.com/fermat_science

Principal activities:

- ANOTHER IDEA OF MATHS
- Fermat Science organizes scientific and mathematical festivals and workshops, lectures, exhibitions, shows, in order to develop another idea of mathematics with the pleasure of discovery. Public affected in 2016: 25 000 people.
- Organization of events around math: scientific workshops, festive meetings on science : Festi'Maths, Récréa'Maths, Mater'Maths, the Science Festival, the Festival of math, math week, the Scientific Book show (exhibitions, workshops, lectures, performances)
- Creation of educational tools: Workshops, exhibitions, educational kits...
- Through the various workshops, Fermat Science offers a concrete approach to mathematics and historical scientific notions. The workshops, based on manipulations and games, offer a practical approach and put the children in a learning process which allows to "see and understand ".

Principal projects:

- Erasmus+ project : MathSpaces
- Europe project: Diffusion des mathématiques en Occitanie
- Opening of La Maison des Maths, a new museum of maths, in Beaumont de Lomagne, 2019

Principal supporters:

- Fondation EDF
- Fondation Banque Populaire Occitanie
- La Région Occitanie
- Le département 82
- La mairie de Beaumont de Lomagne
- Le LABEX CIMI Université Toulouse
- Erasmus +

Il Giardino di Archimede.
Un Museo per la Matematica

Typology:

Non-profit association

Home:

Via S. Bartolo a Cintoia 19a
50142 Firenze

Web page:

http://www.archimede.ms

Contacts:

archimede@math.unifi.it
tel +39(0)557879594

Principal activities:

- Interactive Museum, completely devoted to Mathematics and its applications, in which the visitor learns how to familiarize with mathematics, discovering its important role in everyday life. Museum separated in four sections, independent and dedicated to different themes: from the geometry of curves to Pythagoras's theorem, from Leonardo Fibonacci to the simple machines studied from Galileo.
- Organisation of National Conferences for Maths teachers to include and promote history of Maths in their teaching
- Organisation of Festa della Matematica (every year)

Principal projects:

- Erasmus+ project: MathSpaces
- WELCOME, local project for inclusive Museums
- Le Chiavi della Città, Local project for Schools

Principal supporters:

- MIUR: Minisitero Università e Ricerca
- MIBAC- MInistero per i Beni e le Attività Culturali

IMAGINARY
open mathematics

IMAGINARY gGmbH

Typology:

Non-profit association

Home:

Mittenwalder Str. 48
10961 Berlin, Germany

Web page:

https://www.imaginary.org

Contacts:

info@imaginary.org

Principal activities:

- IMAGINARY creates and distributes interactive exhibits that communicate modern mathematics to the general public
- promotes mathematical education and knowledge worldwide
- offers images, 3D prints, hands-on exhibits, and interactive software as open source on its website
- empowers people to organize local exhibitions and hence carry modern maths to every corner on Earth
- consolidates the international network of math enthusiasts
- is a think tank for modern mathematics communication
- is collaborative, global, free, and close to research.

Principal projects:

- IMAGINARY has inspired millions of visitors in more than 50 countries and 27 languages since 2008.
- Several permanent installations, e.g.:
- National Museum of Mathematics, New York, USA
- Deutsches Museum, Munich, Germany
- Mathematical shopping center “Mathematikon”, Heidelberg, Germany
- CAMP (Center for Application of Mathematical Sciences), Daejeon, South Korea

National traveling exhibitions:

- Belgium, France, Germany, Israel, Netherlands, Russia, South Korea, Spain, Taiwan, Turkey, Uruguay

IMAGINARY gGmbH

Principal projects (cont.):

Numerous temporary exhibitions and events, e.g.:

- NIMS-IMAGINARY exhibition at the International Congress of Mathematicians, Seoul, South Korea, 2014
- Mathematics of Planet Earth exhibition at Imperial College London, UK, 2015
- IMAGINARY exhibition at the Next Einstein Forum (NEF), 2016, Senegal
- IMAGINARY exhibition at Volkswagen DRIVE, Berlin, Germany, 2017

Examples of other projects:

- Erasmus+ project: Mathspaces
- IMAGINARY Conference on Open and Collaborative Communication of Mathematical Research, Berlin, Germany, 2016
- Math Communication Network (newsletters, open resources, information...), supporting the community of math outreach professionals.
- Software infrastructure for the new Astronomy museum ESO Supernova in Munich, in collaboration with the Heidelberg Institute for Theoretical Studies (HITS). It will be opened in 2017.
- Competitions for mathematical images, artworks, exhibition modules, films etc. Including Mathematics of Planet Earth (2013 and 2017), MathLapse filmfestival (2016), Math Creations (2017).
- Workshops for schools and other audiences
- Conference contributions (papers, workshops, artworks, films)
- Snapshots of modern mathematics from Oberwolfach: short texts on aspects of modern mathematics written by researchers visiting the Mathematisches Forschungsinstitut Oberwolfach (MFO)

Principal supporters:

- Mathematisches Forschungsinstitut Oberwolfach (MFO)
- Leibniz Association
- Klaus Tschira Stiftung (KTS)
- Federal Ministry of Education and Research (BMBF)

mmaca
Museu
de Matemàtiques
de Catalunya

MMACA
Museu de Matemàtiques de Catalunya

Typology:	Non-profit association Declared of public interest Official entity for teachers training
Home:	Palau Mercader - Parc Can Mercader 08940 Cornellà de Llobregat
Web page:	http://www.mmaca.cat
Contacts:	contacte@mmaca.cat @MMACA_cat https://www.facebook.com/mmaca.cat

Principal activities:

- Permanent exhibition at Palau Mercader (hands-on exhibits)
- 2 to 4 temporary itinerant exhibitions/year through Catalonia
- Local and regional fairs
- Conferences and workshops: Maths teachers' associations congresses, universities, officials teacher training centres...
- A shop that sells rigorously selected games for their mathematical rating

Principal projects:

- Community of Practice about Formal and Non-formal Education Entities
- Mirrors and Kaleidoscopes exhibition (in collaboration with CosmoCaixa)
- LeonarDomes workshop
- MathWeek in Ireland
- A Pull-on Maths Exhibition in a Suitcase (in collaboration with BCN County)
- A MathLab for Tens Talent for La Caixa Foundation
- Math Centres web in Catalonia
- Organization of 3rd MATRIX Conference (October 2018)

Principal supporters:

- FEEMCAT - Federation of Maths Teachers' Associations of Catalonia
- SCM - Maths Society of Catalonia
- Department of Education of Generalitat of Catalonia
- Department of Education of Barcelona County
- Municipality of Cornellà
- Cellex Foundation
- Hewlett-Packard Spain

1 What is "non-formal" education?

According to UNESCO (Youth, Education and Action to the New Century and Beyond, Unesco, 24 July 1998):

- Formal education: the hierarchically structured, chronologically graded education system from primary through to tertiary, higher education institutions.
- Informal education: the process whereby every individual acquires attitudes, values, skills and knowledge from daily experience, such as from family, friends, groups, the media and other influences and factors in the person's environment.
- Non-formal education: organized educational activities outside the established formal system that are intended to serve an identifiable learning clientele with identifiable learning objectives.

1.1. Differences between formal and non-formal approach.

The differences that Rogers suggests (Rogers, 2005) are:

FORMAL APPROACH (F.A.)	NON-FORMAL APPROACH (N-F.A.)
Selective: a strainer, a pyramid, select first and then train; system rejects participants at various stages; once out, cannot get back in; system ends up with very few (elitist); costly	Open: can get in and out at any time; no prior selection. Only self-selection by participants; no rejection, no failures, no permanents dropouts; cheap
Remote from life: a period of all education and no work, followed by all work and no education; takes participants out of life into full-time education; rejects life experience for classroom experiences; learn now for future use; curriculum academic, irrelevant, colonial	In-life education: learning to be, not learning to become something different; learning how to cope with living now; uses experiences and existing knowledge; relevant curriculum, immediate application; Part-time, not full time; uses indigenous knowledge
Terminal: front-end loading education, "banking approach"; send participants out "trained" for life, fully equipped, no need for more; certificated	Lifelong: education never complete because always coping with new things; no so interested in certificates; admits "I don't know"
Results: creates dependent learners; learning stops when teacher is not there	Results: creates self-reliant, independent and continuing learners.

We consider that this analysis tends to exaggerate the defects of one option and the merits of the other.

Roger might be underestimating the constant attempts at renovation that formal teaching has undergone through the contributions of such different factors as constructivist pedagogy or the technological revolution, trying to answer the challenges of a society in permanent and rapid change.

Convinced that we need to improve the exchange of experiences and good practices between schools and museums/science centres, we have tried to consider the differences between formal and non-formal approach with a little more objectivity and in relation to:

- **Timing**: rigidity to previous predictions. School activities are performed in an allotted time, so flexibility is minimal. In the non-formal approach, we can try to get the maximum of flexibility and each participant can stress the time necessary to perform a task in relation with interest, acknowledgment or ability.
- **Contents**: fidelity to the curriculum. The non-formal approach is often based on disciplinary contents that are not included in the school curriculum. Some people working in science centres consider that their entire proposal must be extracurricular, so that a visit to a museum could be an original cultural experience, absolutely different from a school activity.
- **Dynamics**: individual/collective tasks, results, verifications… In schools, the average student progress often determines the pace of the class, condemning those who are not equipped to be left behind and those who have more will and/or capacity to be frustrated. In Science Centres' exhibitions, each participant, even though during the task a cooperating group has been formed, decides when to stop. A characteristic of non-formal education is the heterogeneity of the groups that are formed, sometimes around a single exhibit. The result is a more "real" and less predictable communication.
- **Relationships**: transmission (from one who knows to one who doesn't know) and collaboration (community of learners). In a non-formal approach situation, the roles are not previously assigned; the exchange of information is free and bi-directional; the authority is earned on the field and is still temporary.
- **Communication**: quantity and type of information necessary to perform the task. In school, student performance is supported by a big amount of information: professor, text-book and supplementary means. In a Science Centres' exhibition starting information is often reduced to the simple rules needed for developing a task. Facilitators should help as little as possible; to suggest, never to solve; to accompany, never to guide. Deepening or enrichment of notions is delegated to another situation, like a workshop (in the museum or at school), individual research or -why not- to a later school-based intervention, a bridge with formal education.

Exhibits provoke spontaneous communication between users.

There are two kinds of people: those who love maths and those that have not yet found out they love it.

MMACA

Of course, our interest, the experiences and proposals that we advance are centred in the learning of mathematics. We believe, however, that many of them can be extrapolated to other disciplines, as STEM or STEAM reinforce projects seem to demonstrate.

> ***Mathematics is a game played according to certain simple rules with meaningless marks on paper.***
>
> ***David Hilbert***

In fact, mathematics presents a relevant formalism, definitely indispensable to treat certain complex theoretical aspects. However, a premature and excessive formalism may cause form and language to become the main content of the discipline, thus excluding a certain natural intuition and a playful pleasure, which is also so typical of the human race.

The final risk is that a consistent part of people is taken away from mathematics. They will form a distorted idea of mathematics as a cold and dry discipline, far removed from reality, incomprehensible and abstract.

> ***We don't stop playing because we grow old; we grow old because we stop playing.***
>
> ***George B. Shaw***

We are not talking about «fun science» [1] , but a seductive approach to bring more people to have a positive relationship with STEMs. Also when:

> ***Humour is by far the most significant activity of the human brain.***
>
> ***Edward de Bono***

1.2. Common elements of formal and non-formal approach

This is the point: we deeply believe that formal and non-formal learning approaches must collaborate. In a museum or a science centre we must not repeat school mechanisms. We should not teach, but offer open learning situations. Often, a visit to an exhibition provokes a positive emotion (hearts-on) that can be the seed of a new interest on the STEAM.

In schools, we must take advantage of every opportunity that allows students to develop on their own interests and abilities, alone or in a group, and encourage them to make their investigation available to the class (minds-on).

Furthermore, there are important aspects that must be developed both in school and science centres.

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1 http://www.ecsite.eu/activities-and-services/news-and-publications/digital-spokes/issue-13#section=section-indepth&href=/feature/depth/fun-science-seductive-science

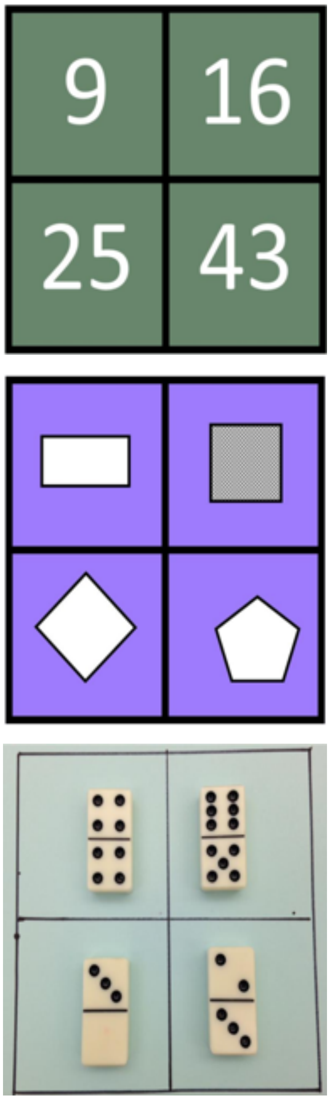
1.2.1. *You can teach nothing to a human being; you can only help people to find the answer inside them.*

Galileo Galilei

A significant and long-lasting learning cannot be separated by a powerful desire to learn on behalf of learners. This means that we have to offer activities that stimulate interest and autonomy, that encourage them to look for and to communicate the results of their work.

We can propose open problems, in which more than a solution is possible and strategy can be discussed. A good example is this Californian open-source page, ***Which one doesn't belong?*** [2]

Open challenges allow users to find an original solution or a strategy we may have not foreseen. That brings a deep personal satisfaction that we have all felt ourselves, sometimes with materials we have known for years.



Which one doesn't belong to the group? Why?

The goal of promoting personal/group investigation can be pursued through computer supported activities too. Starting from a standard situation (a shape, a drawing, a table of data or a formula, depending on launched program), little changes can be introduced and observed in real time as the output changes. The immediacy of the response facilitates the development of the investigation. It is easy to observe how the sequence of learning action described by Kolb's circle (see section 1.2.c) works in computer supported activities too.

In the same line, it could be interesting to show some of the historical solved (recurring fractions, sum of infinite sequence of square or triangular numbers…) and/or unresolved problems (for ex., Fermat's conjecture, the squaring of the circle, the trisection of the angle ...), accompanied by physical or virtual hands-on materials that highlights the simplicity of the utterance and the difficulty of the solution.

1.2.2. *The principal goal of education in the schools should be creating people who are capable of doing new things, not simply repeating what other generations have done.*

Jean Piaget

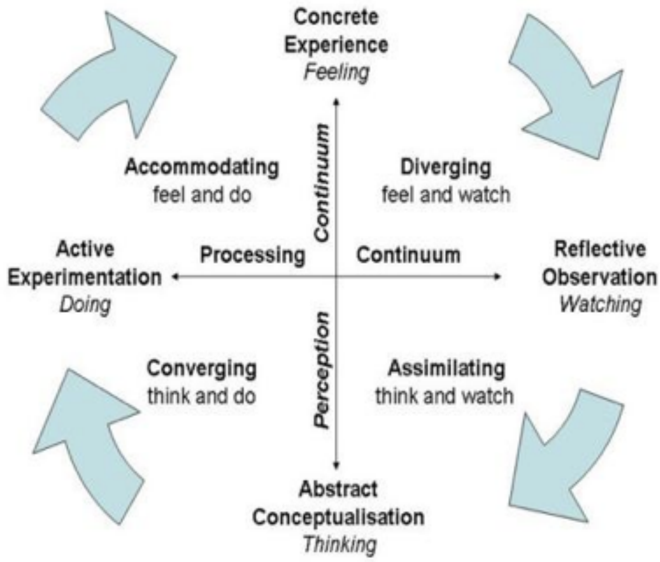
We should definitely opt for a competency-based learning; the mere transmission of content and skills is not enough.

2 http://wodb.ca/index.html.

> ***«Competency-based learning is learner-focused and works naturally with independent study and with the instructor in the role of facilitator. This learning method allows a student to learn those individual skills they find challenging at their own pace, practising and refining as much as they like. Then, they can move rapidly through other skills to which they are more adept»***
>
> ***https://en.wikipedia.org/wiki/Competency-based_learning***

1.2.3. There are different ways of learning, so we must try to offer alternative approaches and experiences.

Kolb's circle is one of the most popular models that show the relationship between experience, reflection, conceptualisation and experimentation.



There are people who need to start from a concrete situation, a motivating challenge to put in motion a personal learning process, which should cross the four phases to be consolidated and to introduce a new investiagation or experience. But, at our exhibition we also met users who need to know the theoretical scope to start to interact with an exhibit.

As told, each person must find their own way of learning and chose the best way to begin. Then, it is important to cross all the phases, helped by the educators or collaborating with other learners.

In some science centres, we often offer a hands-on experience, a challenge to solve. Of course, handling objects is not our goal, but just the beginning of a process that must conduct to building concepts, to be eventually reinforced through active experimentation. The phases could be:

Handling objects ⟶ building objects (workshop) ⟶ building concepts

At the same time, we must ensure that our exhibits prove challenging on different aspects.

They must be:

It means doing, thinking and feeling not in a sequential way, but in a virtuous circle.

1.2.4. *By giving our students practice in talking with others, we give them frames for thinking on their own.*

Lev S. Vygotsky

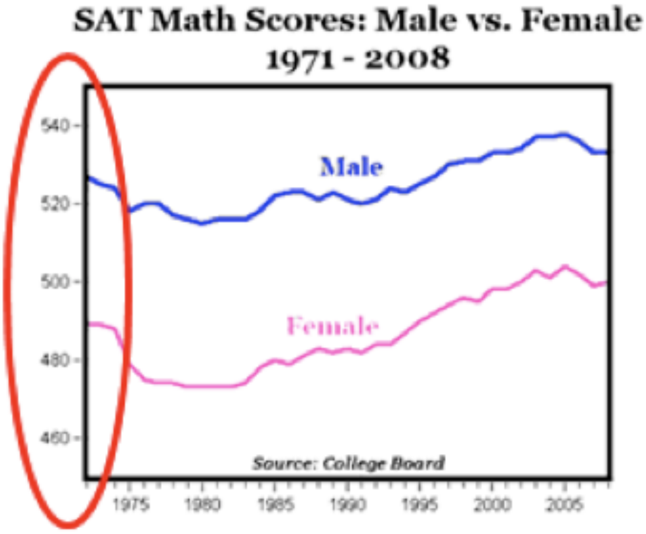
Communication is one of the main components of competency-based learning and, of course, of collaborative dynamics.

We see every day how groups of 3-5 people, who do not know each other, are formed spontaneously around an exhibit to collaborate in the solution of the challenge. Heterogeneous groups in sex, age and abilities converse, propose, verify hypothesis and try solutions.

1.2.5. *Science is not a boy's game; it's not a girl's game. It's everyone's game. It's about where we are and where we're going.*

Nichelle Nichols

In our exhibitions and workshop we do not detect difference neither in interest nor in performance between girls and boys. The gaps that some investigations complain about (not always in good faith) have no significant evidence, not even among teenagers, where some dynamics and attitudes are more conflicting. The purely collaborative nature of the activities and the importance of the communicative aspect favour the participation of girls, who often play a leading role. Competitive activities are few and always oriented to improve personal performance, not at the overcoming of the opponent. So, understanding the roles of the game and the collaboration necessary to elaborate and exteriorize a winning strategy are the principal goals, in which all users can collaborate and self-esteem can grow.



Scale seems choosed to amplify the difference.

1.3. Modes in which a non-formal approach to learning can be implemented

This table schematically summarizes the actions that, from non-formal education, can be addressed to formal education centres to increase and improve learning.

ACTION	TIMING	DYNAMICS	CHARACTERISTICS	
EXHIBITIONS	60 to 90 min.	Free or guided activities.	Experience	Excitement
			Surprise	Seduction
FAIRS	10 min. to 3 h	Addressed activities.	Games	Fun
			Competitiveness	Crowd
WORKSHOPS	60 to 90 min.	Guided activities	Accompany and complement the curriculum.	Building experiences and materials.
SPEECHES	10 to 90 min.	Participatory activities.	Surprise	Magic
			Curiosity	Show
TRAINING TEACHERS & EDUCATORS	3 to 20 h	Guided activities	Theory	Appropriation
			Deepening	Empowerment
			Peer tutoring	Creativity

- **Exhibitions**: different models are possible, from a guided visit to something that is more like a free workshop. In the first case, the educator leads the group to discover the key concepts, highlighting the most significant exhibits.

  In the second modality, the facilitators only explain one exhibit, just to give a model to approach and use the exposed materials. Then they let visitors freely interact with the exhibition, minimizing their interventions. They can suggest, not solve; they can adjust the dynamics or recommend new challenges.

- **Workshops**: often represent a bridge between exhibitions and schools, because the facilitator can drive the visitor to the conceptualization.

  Workshop can be improvised in the exhibition, linked with an exhibit, to deepen a theme that has aroused interest and to promotefurther investigation.

  More traditionally, they can be done in a classroom, in order to reconstruct materials and to better understand its meaning and mathematics that usually hides behind the easy approach we give to our exhibits.

  Inspired by the Exploratoriun of San Francisco's experience, an interesting debate is open: What are the substantial differences between a school and a museum workshop?

- **Fairs**: the goal is to show the most playful and pleasant side of mathematics. The activities must be fast and attractive. It would be a big success if the students want to organize a school fair, performing themselves as facilitators.

- **Speeches**: It's a good opportunity to consolidate the presence of a museum or a Science Centre in the territory, showing surprising aspects of mathematics, such as, for example, its deep relations with magic tricks. Conferences also represent a wonderful opportunity to create a space for dialogue - as necessary as unusual - among educators, cultural workers, creatives and researchers.

- **Training courses for teachers and educators**: A non-formal approach to learning is very useful in this situation, when sharing experiences and co-elaborating strategies is the main goal. Teachers need to master materials and activities, to control the mechanisms to eventually adapt them to their class situation. It is a paradigmatic situation for implementing competencies skills and stimulating the construction of their own mathematical concepts.

  The debate in this area is very open:

  - Training for activities during the visit (short, at least 3 hours) or to start a collaborative museum school (long, at least 20 hours).
  - Possibility to intervene in the organization of initial teacher training.
  - Communion of the training of teachers and museum educators.

2 The benefits of learner engagement

To begin this chapter, let us use some concrete examples of learning from everyday life to make a simple comparison to answer our question.

2.1. Does school kill creativity? [1]

It is often revealing to talk with 4-5 years old children about what they learn at school. You would be amazed to hear them use precise vocabulary to tell these discoveries, count, declaim a nursery rhyme, talk about the evolution of humankind, the human body, … And yet, these children have no notebook in their bags and return home with no homework. Going into a kindergarten classroom enlightens us: the walls, the tables, the shelves are covered with materials, panels allowing children to see, hear, touch, feel and even taste.

Creativity work
La Maison des Maths

In a kindergarten classroom, the five senses are stimulated [2] [3]. Indeed, some investigations in neurosciences, and in particular Neurolinguistics Programming, suggest that the learning process is more effective if multiple senses are stimulated , . Kindergarten teachers regularly use this principle. For a child of this age everything is playing. Discovery through manipulation is the vector of learning.

Does it work if we consider abstract concepts learning (such as some mathematical contents)? Maybe we are focusing on previous approaches to reiterate that experimental investigation is important in school.

Some authors point out that high school students are less creative [4] and difficulty in solving complex tasks grows during high school… Does school kill creativity?

2.2. Music and mathematics.

Learning music is part of formal learning. Indeed, the execution of a music score requires assuming the codes of deciphering it: solfeggio. Whatever instrument is chosen, the bases are fixed and you only have to look for the

1 http://www.huffingtonpost.com/line-dalile/a-dictator-racing-to-nowh_b_1409138.html
2 Stein BE, Stanford TR, Rowland BA (December 2009). "The neural basis of multisensory integration in the midbrain: its organization and maturation". Hear. Res. 258 (1-2): 4–15.
3 Stein BE, Rowland BA (2011). "Organization and plasticity in multisensory integration: early and late experience affects its governing principles". Prog. Brain Res. 191: 145–63.
4 https://people.goshen.edu/~marvinpb/11-13-01/Effects-of-Stereotypes.html
http://www.sheknows.com/parenting/articles/1024783/kids-are-getting-smarter-but-less-creative
http://faculty.ucr.edu/~aseitz/pubs/Shams_Seitz08.pdf
https://www.mpg.de/8934791/learning-senses-vocabulary

right way to produce a C, D, or F sharp...The formal system of the music score will be highlighted using a tool of concrete representation: the instrument. An early contact with the instrument, previous at the exact coding of the score, could be considered a non-formal approach to music. Maybe it does not increase the number of professional musicians, but the number of young people who can appreciate music.

Diversification learning
La Maison des Maths

The manipulation of materials and the emphasis on the stimulation of the senses refer to the importance of having adequate tools to concretely represent formal systems.

A non-formal approach to mathematics learning is based on functionalizing mathematical knowledge, let it appear as reasonable decisions, formulations or arguments. It leads to creating situations in which students construct mathematical knowledge without first being taught directly and formally.

Non-formal mathematics raises the problem of why and how to collect knowledge and organize it into a hierarchy to facilitate its use, deepen learning and adapt it to other situations.

2.3. Is the formal system the only way to learn?

The test-error method is very important for children's learning and crosses the history of mankind, which has discovered, handled and modelled concepts. So, why in our Western countries, do we continue to believe that formal learning is the only way to knowledge?

In most Western schools curricula, mathematics occupies a prominent place and its learning is designed on this model. Perhaps more than any other discipline, it appears as a scientific teaching that transmits objectives, universal and fundamental knowledge. The so-called formal and/or academic mathematics is considered a must-have piece of equipment that every pupil must acquire and almost basically master at the end of compulsory school. Add this to the culture of quantifiable results, the examination of knowledge, certifications…

The maintenance of these cultural beliefs is fallible: this is shown in surveys on school achievement and, in the case of mathematics, by our learners' results compared to those of other OECD countries.

PISA 2015 provides results from 540 000 15 years old students in 72 OECD countries and non-OECD partner countries. A panel that is considered representative of the 29 million pupils of this age schooled in these 72 countries and economies. The above classification is reproduced as presented in the PISA study. It is based on points acquired in science, the "major domain" of the study.

#	COUNTRY	MATHS SCORE	#	COUNTRY	MATHS SCORE	#	COUNTRY	MATH SCORE
1	Singapore	564	6	Macao (China)	544	15 ex	Germany	506
2	Japan	532	7	Canada	516	20	Belgium	507
3	Estonia	520	8	Vietnam	495	26 ex	France	493
4	Taipei (China)	542	9	Hong-Kong	548	28 ex	Spain	486
5	Finland	511	10	China	531	34	Italy	490

This list puts together countries with very different social and economic situations, making comparisons unfair, so that these results should be taken with some caution. In any case, it forces us to ask ourselves the question of the stagnation in the formal learning of subjects. Of course, it would be unthinkable not to teach contents, methods or to train specific abilities, but reproducing obsolete models of education is suicidal.

Could this difference in results between countries be the result of different and more effective learning? What do the countries that achieve the best results do?

2.4 - What is the best way to learn?

According to Rocard's report "Science Education: Now" [5] and the Eurydice report "The teaching of mathematics in Europe: common challenges and national policies" [6], an effective teaching of mathematics includes the use of various teaching methods.

EU reference documents about learning

There is a general consensus that certain methods such as problem-based learning, investigation and contextualization are particularly effective in achieving results and improving student's attitudes towards mathematics. Although most central authorities in Europe say they provide some guidance on how to teach mathematics, there is a need to strengthen support for methods that encourage active student participation and critical thinking.

It is obvious, of course, that as in the example of learning music, having assimilated the codes of deciphering maths (recognition of numbers, basic operations…) is an asset. But what about self-taught music? Paul McCartney is not known to have attended formal music schools. Does it make him less competent in sound, rhythm, measure…?

Other popular musicians have a solid formal background, so that all options remain open and very often intertwine.

In any case, it is worth pondering how the advent of technology, even simple, from a calculator to a text corrector, has changed the value of the essential components to good performance. The most technical and mechanical aspects have been delegated to the machine, while human beings are essential for creativity, conceptualization and, last but not least, product quality control, also facilitated by the speed at which the machines provide outputs.

In the field of music creation, there are programs (as Sibelius, Finale NotePad, Musescore…) that, through a heuristic dynamic, transcribe compositions made by ear on a musical staff, enabling immediate, albeit mechanical, listening and necessary adjustments.

5 http://ec.europa.eu/research/science-society/document_library/pdf_06/report-rocard-on-science-education_en.pdf

6 http://eacea.ec.europa.eu/education/eurydice/documents/key_data_series/134EN.pdf

The best technology (interactive, open, communicative programs) has interesting pedagogical aspects. Part of human creativity resides in breaking with patterns [7]. The ability to talk about something that does not exists or to lie, reductio ad absurdum demonstrations or proof by contradiction, the overthrow of theoretical bases ... can crank with the rigidity of the constraints typical of mechanical performance. So, increasing challenges and coping with consistent tasks can enhance the acquisition of technical skills by the professional or amateur musician.

2.5. Non-formal mathematics, at the heart of learning.

Exhibits
La Maison des Maths

Many studies (e.g. Carraher & Schliemann) [8] [9] [10] reaffirm the idea that non-formal mathematics can provide a basis on which learners can rely to build more sophisticated mathematical knowledge.

These two authors consider that classroom activities can be transformed so to allow the learner to experiment with a plurality of mathematical situations, tools and concepts that make the links between the mathematics of everyday life and those developed at school explicit.

With non-formal mathematics, learners are at the heart of learning: they discover, handle and model. Non-formal approach can be based on individual and group learning as part of an overall collective improvement. It is participatory and learner-centred, and action and experience-based.

Advantages of this model are numerous and even exceed the mathematical acquisitions, by touching on capacities of cooperation, creativity, freedom, communication…

2.6. A must for our society.

In conclusion, working with non-formal mathematics responds above all to a societal need to motivate young people, without discrimination of sex, income and origin, in the STEAM branches, that are useful for the maintenance and development of our societies.

Motivation and self-esteem grow because one can choose which challenge to solve, alone or in group, thinking, discussing, explaining, using the time they need, whether to limit themselves to the resolution of the challenge or to deepen the study of the subject.

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7 We have to learn the rules well, so we can break them in the right way. Dalai Lama.

8 Carraher, D. & Schliemann, A.D. (2002). Empirical and Logical truth in Early Algebra activities: From guessing amounts to representing variables. Symposium paper NCTM 2002 Research Presession. Las Vegas, Nevada, April 19-21. [View abstract]

9 Schliemann, A.D. & Carraher, D.W. (2002). The Evolution of Mathematical Understanding: Everyday Versus Idealized Reasoning. Developmental Review, 22(2), 242-266. [View abstract]

10 Carraher, D.W. & Schliemann, A.D. (2002). Is everyday mathematics truly relevant to mathematics education? In J. Moshkovich & M. Brenner (Eds.) Everyday Mathematics. Monographs of the Journal for Research in Mathematics Education, 11, 131-153.

Mathematic tales
La Maison des Maths

By activating the principle of experimental trial and error, the learner can, in some respects, become self-taught in mathematics. Universal activities related to mathematics (counting, measuring, situating in space, drawing and building, playing and explaining) can be developed according to different cultural models.

According to constructivism, each user starts the process of conceptualization of mathematics and abstraction.

This first step prepares the teacher's intervention, which aims to systematize the acquired knowledge, propose generalizations, formulae and algorithms, and analyse theorems.

It can go on looking for relationships with other not scientific disciplines, application and enrichment of the contents.

It is thus possible to weigh the Pythagoras theorem, to access a mathematical vocabulary by slaloming around cones or by launching a spherical or ellipsoidal balloon, to draw symmetrical patterns, to measure with respect to the entrance of a palace of the Thousand and One Nights…

> ***The future belongs to no one. There is no precursor, there are only latecomers.***
>
> ***Jean Cocteau***

It is up to us to use the lessons of our history, our experiences as little children, our personal discoveries to live and make the joy of mathematics live in the very near future.

3 The facilitations for the school system

These last years, the question of the importance of skills in mathematics has gained weight and presence. These skills are now considered necessary for self-fulfilment, citizenship, and social inclusion.

The school world cannot stay out of this debate or avoid questioning the way in which STEMs and maths are specifically dealt with in formal education.

According to Anne Siéty, a French educational psychologist:

> ***Some students in trouble suffer from seeing in mathematics a subject they are not involved in and which is of no concern to them.***

The first experiences of children are primordial, but students are often afraid of mathematics and some of them modify their educational career to avoid them. Different approaches can improve their attitudes so they can feel the taste of discovery again and thus increase the intellectual level in mathematics and open new and more satisfactory possibilities of learning.

According to Céline Alvarez, linguist – author of Les lois naturelles de l'enfant (The Natural Laws of the Child):

> ***Nature encourages children to learn in the most powerful way possible: by giving them an intrinsic desire to learn more. Children do not need formal and masterful explanations. They need to live and confront the continual succession of shocks that this experience in the world will offer them***

Recently, we have seen a change in the way we teach mathematics, assuming a non-formal approach to this discipline, with more research activities and fewer exercises with unclear interest.

The approaches and methods used to discover mathematics can have a significant impact on how students learn in the classroom as well as on the quality of their learning. They can improve students' understanding and help them know about mathematical rules and procedures. This method also influences the pleasure that students get from learning, in both quantitative and qualitative terms.

3.1. In mathematics, is non-formal learning complementary to formal learning?

3.1.1. A necessary support for both approaches.

Fermat Science

Children like to learn if they are supported. According to Céline Alvarez:

> ***Children have an extremely powerful learning software but they need ... a guidance from someone more advanced than them, who can advise about the important elements to observe and take into account to move on.***

Teaching methods can, for example, focus on mathematical principles and processes or on the application of mathematics in real life. But they also determine the nature of interactions, including those between the facilitator and the whole group of children, between the facilitator and each individual child, or between small groups of children.

The facilitator must also encourage positive exchanges and understanding between children and set the limits of a structuring and safe framework so that each child can develop the sense of mutual help that is natural from an early age.

It is also necessary to change the attitude of adults and start from the principle according to Anne Siéty that:

> ***Every student is intelligent. If someone has not been able to understand such and such point, it is because an imperative reason prevented him from doing so.***

Then, it is up to adults to find the right tools to help the child overcome obstacles. That is why it is necessary to propose several kinds of approaches for the child to succeed. Mathematics should not only be based on repetitive learning but on the pleasure of discovering one's own intelligence. Success is a powerful motivation to work harder.

3.1.2. Non-formal learning and playful mathematics [1]

Fermat Science

Putting the fun back into mathematics or making mathematics playful does not mean making it simpler, reducing the level. It is amazing to see how playing helps assuming complex notions. It makes it possible to give an immediate appeal to mathematics and incite people to deepen the study of the discipline and thus to accept the rules and the more technical aspects easily.

1 Viladot, P.; Stenglen, E.; Fernandez, G. – "From fun science to seductive science" –Spokes #13, nov. 2015

Fermat Science

Playful mathematics replaces obligation with instruction. Children are not bound to do something; they just have to follow a game rule. According to Stella Baruk [2], professor of mathematics and researcher in psychology, they realize very quickly that good understanding of the rules leads to success. To a student in a playing learning situation, it is a matter of comprehension, according to the etymology of this term: *cum prehendere*, to take with oneself. This way of de-dramatizing the learning of mathematics then allows us to painlessly return to a more formal learning.

The notion of game is very wide, it can range from a traditional game, to a magic trick, to origami, puzzles or the construction of strange objects. Playing is a good way to approach maths.

a. An example of a playful workshop: Tangram

Fermat Science
Tangram

Tangram can be used to develop the sense of observation and introduce young people to geometry in a visual way.

The origins of the game date back to the 7th century in China: legend tells that an emperor, admiring a magnificent tile of earthenware, would have accidentally dropped it on the ground, where it would have broken into seven pieces. Attempting to reconstruct the broken tile, he was never able to succeed and recreated instead thousands of different figures. The game of Tangram was imported to the West only at the end of the eighteenth century.

The rules are simple: after having followed a construction program to create one's pieces of Tangram, the instruction is to make silhouettes representing characters, geometric figures, animals, letters... There are a great number of possibilities; there are some 2000 geometric or figurative models more or less complicated to build.

The use you can make in mathematics is impressive.

Starting from a leaf of paper, folding and cutting the different parts, you can obtain, one at a time, the seven shapes. This path allows each shape to be analysed and compared with previous ones. The teacher can conduct the observation: asking and suggesting. Comprehensive School curriculum of descriptive geometry can be introduced or verified in about an hour.

Then, if you want to go deeper, other activities are possible for different school steps:

- Representing numbers (assuming the smallest shape as unit)
- Representing fractions (assuming the primitive square as unit)
- Building convex polygons with the seven shapes (demonstration of why only 13 polygons can be made is a very interesting activity for older students [3])

2 https://en.wikipedia.org/wiki/Stella_Baruk
3 http://archive.bridgesmathart.org/2012/bridges2012-553.pdf

- Building squares and/or rectangles with 1 to 7 shapes.

It is important to underline that all these activities are open problems, sometimes unsolvable, often with more than a solution, so that a discussion between the students can be opened, enhancing understanding of concepts and communicative skills.

Fermat Science
Geometry and History

b. Another approach: the history of mathematics

The history of mathematics makes it possible to understand certain mathematical notions by placing them in their context. This approach makes learning meaningful and richer. Instead of detached notions of life, it places mathematics in the evolution of humanity.

Using the history of mathematics can also become a good way to create motivation by telling mathematical discoveries as part of the human adventure. This makes it possible to give those who discover them the desire to better understand, for example, Thales and the measurement of the pyramid, Eratosthenes and the measurement of the circumference of the Earth, or the problem of the Brachistochrone curve, which involved a dozen of the most famous mathematicians through a couple of centuries.

3.1.3. Synergy of these different forms of learning

Opinion on education has evolved. Considering the definitions of formal and non-formal learning, it is clear that education must be a continuous process combining these approaches.

a. Why using these approaches?

It is the inadequacies of the “classical school system” that have aroused this increasing interest in non-formal forms of learning as a complement to the school. In fact, this interest is explained by the advantages of non-formal approach for socio-economic and socio-cultural development, especially after the disappointing results of a school model whose educational content is ambitious but neither practical nor concrete, often away from students’ everyday lives. The costs remain high and therefore limit its expansion. It is these limitations of school education that make it necessary to use more and more out-of-school activities.

b. How can this non-formal learning complement school actions?

In non-formal learning, participants take an active part and are involved in the learning process, with direct consequence in the information provided. Regardless of content, knowledge and skills, non-formal learning provides the opportunity to develop autonomy and participation. In this context, mutual assistance is essential. It is based on a spontaneous ability of the child to exchange with others, an innate tendency that must be developed for the greater benefit of all. According to Céline Alvarez:

No teacher can compete with the ease of knowledge transfer between children of different ages: the fascination of a 5-year-old child over a 3-year-old is exceptional, as is the spontaneous enthusiasm for helping children who need it.

This implies the introduction of good quality materials that are attractive and able to wake up the curiosity of children and make them want to confront the difficulties of learning.

This also implies revising the status of error as defined by, among others, Agnes Rigny, and Anne Siéty. Making mistakes in a phase of experimentation is not a problem. The result is examined in relation to the expected result. If it is not being reached, the next step is to understand where the mistake is and to start again from a new hypothesis. This method produces real and lasting knowledge, when the alternative is applying rules imposed by adults, learned without being understood.

Active learning allows the students a deeper self-knowledge, teaches them to analyse themselves while being accustomed to taking initiatives within a group and measuring their impacts.

It is this participation process that makes non-formal education a formidable school of citizenship. There is no standard imposed, no obligation to answer in a limited time and no sanction. Personal development is respected. The challenge is not to get a good mark, to perform and leave, but to take pleasure in the discovery, so to feel the satisfaction of overcoming an obstacle and access knowledge.

3.1.4. Take away the risk of inequality

We see an increase in private schools and universities that meets the needs and wishes of privileged students (and their parents), while public education remains essential for the majority of the less privileged. The gap that may increase between public and private schools poses a threat to both formal and non-formal education.

Moreover, for students who are socially distant from culture, the mathematical language can be a source of misunderstanding, generating failure and stress because there is sometimes a discrepancy between mathematics and reality. If mathematical formalism becomes the essential element of learning, it can only reproduce those social mechanisms that have made maths one of the major class selection elements. Opting for a closer approach to reality does not mean giving up on a correct language or abstract concepts, but improving its development and showing its meaning and utility.

To take away the risk of inequality, it is necessary to develop:

- Non-formal activities accessible to all and everywhere outside schools
- The training of teachers and mediators in the different methods of mathematics education and mathematical language.

3.2. Successful results

3.2.1. Student Motivation

The level of motivation to learn mathematics is an important determinant of student achievement. National strategies to increase student motivation are in place in almost half of European countries .

Indeed, positive attitudes towards mathematics and self-confidence in mathematics learning go hand in hand with better results in mathematics.

Motivation influences decisions about participation in different studies in which mathematics is an important subject: attitudes can affect students' education and career choices.

Research on the main factors influencing students' positive attitudes towards mathematics suggests that teaching methods should be stimulating, diverse and related to students' everyday lives. Non-formal educational approaches thus create the conditions necessary to improve student motivation and performance in mathematics.

3.2.2. Improvement of the grade level

Increasing results is not an immediate goal, but it is conceivable that self-esteem and mastery of more skills and tools will positively modify the approach to mathematics and improve the performance.

3.3 - How to coordinate these different forms of education?

3.3.1. Approach to non-formal learning in schools

The use of games, hands-on materials and other experiences of non-formal approach to mathematics is quite feasible in class.

It is not only a matter of remedying difficulties. It's also to help students to construct and appropriate mathematical knowledge by articulating "regular" sessions and game sessions as closely as possible.

The teacher needs to evaluate the evolution of dynamics and results to organize the different phases and introduce new themes, challenges and proposals.

Game sessions are not a reward after a "real effort", reserved for the fastest or the best performers. They have their full role and the whole class participates in it at the same time. Here, the goal is not to play but to learn through handling, giving another status to error, using concrete notions to gently go towards more abstraction.

Teachers can take the tools available to them by the actors of mathematical culture and mathematical museums or they can create workshops and little exhibitions in the classroom themselves.

3.3.2. Actors of mathematical culture and mathematical museums

Experiences with non-formal education outside the school environment are often poorly understood, especially as they are the work of non-governmental organizations or local associations. There are structures, associations, and mathematical museums all over the world that provide schools with educational activities and tools using non-formal methods of mathematics education.

To discover them, you can visit the database of all these organizations:

http://www.mathcom.wiki

3.3.3. A political choice: an example of proposals in France.

In December 2014, Najat Vallaud-Belkacem, Minister of Education of France, presented the *Mathematical Strategy*, which should improve the level of students in this subject. Ten key measures were announced around three main axes:

• Mathematics programs in tune with their time,

• Better-trained teachers for the success of their students

• A new image of mathematics

In this last axe, we find the Action 7: The promotion of an environment more favourable to learning. The playful dimension of mathematics and the use of digital media will be developed to motivate students more and to encourage their autonomy. The place of game in teaching mathematics, particularly at school will be strengthened. Details of the project:

http://www.education.gouv.fr/cid84398/strategie-mathematiques.html

4 Outside of school. A mind opening.

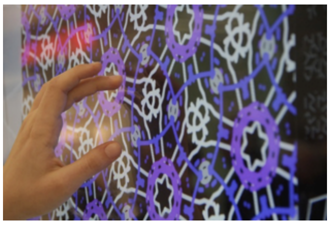
We live in the age of information. Before the existence of mass media (popular journals, radio, TV, Internet…), the sources of information, and the access to knowledge, were essentially two: books and expert teachers. For more than two centuries, societies all over the world have agreed that universal education is a desirable goal, and the school institution is the most efficient system that society has invented to achieve that goal, to transmit knowledge, values and formation from one generation to another. Teachers and books are precious resources and their impact is maximized with this structure. At school, a small group of experts and a supply of books can educate children at large scales. With today's technology, however, information is abundant, ubiquitous, instantaneous, and virtually free.

But of course, information is not knowledge. Humans still need to digest information in an ordered manner, structure the ideas in their brains, learn to be critical, apply concepts and develop skills. Noise and useless information obstruct the emergence of ideas, and the huge stream of stimuli competes for the attention and time of children and adults. Thus, the other precious resource of knowledge and savvy: expert teachers, remain a scarce asset, and schools and universities are still, and probably will for a long time be, the main structure for the education and transmission of knowledge to new generations.

Submissions for an international SURFER competition, created by school children.

Scholar education must be formal and generalistic, give a solid base to prepare students for the next steps of their education. It must be systematic and exhaustive on its methods, so successful students develop the desired competences and get a good command of the subject. In mathematics, this means that formal education has to be vertical, one subject must be consolidated before the next one is built on top, and no important block should be skipped. While this is undeniably effective in its purpose, it necessarily misses a focus on a more panoramic, or horizontal, view of mathematics. Questions related to motivation, applications, philosophy of mathematics, current trends, historical development, etc. are not always properly addressed in the vertical scheme. However, these should also be part of the culture and literacy of well-formed citizens. A non-formal setting, outside the school, can improve this situation by giving a complementary horizontal view, using all the resources available today. Amongst the resources, the human component of math communicators, facilitators in exhibitions, which are possibly the same teachers and professors that also participate in formal education, must not be forgotten. We will discuss, with profusion of examples, some of the aspects of mathematics that can be treated in a non-formal context.

4.1. Abstraction.



The program Morenaments allows to draw tilings using the 17 symmetry groups of the plane.

A fundamental idea in mathematics is abstraction. Not only as opposed to "concrete", but as to the extraction of a fundamental principle that governs the object of study. This fundamental principle is often shared with other objects that would be unrelated at first glance. We will use the concept of symmetry, which is widely exploited on math exhibitions, as an example to describe different levels of abstraction and how they can be addressed in and outside of school.

Symmetry arises in repetition patterns, in flowers, animals and other natural phenomena, in crystals and chemistry, in arts like painting and sculpture, in many musical works, in uncountable games and brain teasers… Many math museums and even amusement parks and other recreational facilities have developed exhibits with a mathematical focus around mirrors, kaleidoscopes and other optical games [1]. With kaleidoscopes we can generate an infinity of images that copy an object by reflection on mirrors. However, all these images share the same structure, the same symmetries, which are determined by the positions of the mirrors and not by the object reflected.

Some exhibitions use software [2] that allow to generate all the 17 bi-periodic tilings of the plane through translations, rotations and reflections of any drawing made by the user. Again, the possibilities of the drawing are infinite, but the structure of the symmetries can be only one of 17 possible [3]. Exhibitions usually include modules related to polyhedra, where the idea of symmetry is also presented.

Symmetry is often presented as a geometric concept, but from a mathematical point of view, it can be described with the more fundamental idea of group as an algebraic concept. The concept of group is quite advanced, not because of the difficulty of its definition, but due to the abstraction required to imagine and manipulate this algebraic structure. Groups are introduced usually on a first or second year course on basic algebra at the university, for maths and science students. After the definition and first examples, one advances through theorems and proofs such as the Lagrange theorem for the order of an element, or the classification of abelian groups, and one learns to manipulate and deduce properties on these structures. This is, of course, a formal learning of advanced mathematics, and it can be easily agreed that this is only for people seeking a higher education in mathematics.

1 Such as Museu de Matemàtiques de Catalunya, Matemilano, Le labosaïque, Mathematikum, and many others.
2 For instance Morenaments, GeCla or iOrnaments.
3 There is a beautiful proof that there are only 17 of these groups using topology, orbifolds and the Euler characteristic. The elements of this proof are presented to a general audience in GeCla, by Atractor association (see the DVD Symmetry – the dynamical view by this association). This subject is usually not taught in school nor even in the regular university math studies, only in specific graduate courses on geometric topology.

The crucial point here is that the idea of a group, a structure of elements that can be combined to produce new elements with very basic properties, is a concept accessible to anyone with a bit of curiosity, almost independently of its mathematical background education. The symmetry structure of a kaleidoscope, a plane tiling, or a regular polyhedron can be identified as a particular group, named and labelled, and this group governs the symmetries of that particular kaleidoscope or tiling but another kaleidoscope or tiling may be intrinsically different because its group of symmetries is different.

This idea can be conveyed without the need of even using the word “group” or stating its definition (although it is the facilitator’s task to identify the audience background and decide how much information to expose). The topic of symmetries of the plane appears in some school textbooks, and chemistry students may have seen the structure of crystals in high school and be familiar with crystallographic classifications. The message on an out-of-school context is that symmetry is not dependent on chemistry, art, or polyhedra, but rather it is an abstract framework that explains all of these phenomena.

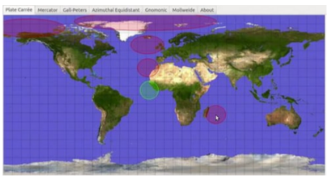
4.2. Modelisation.

Mathematics gallery at the London Science Museum.

The idea of modelisation is in some sense dual to that of abstraction. Modelisation is application: the practical use of mathematics to solve, or at least to understand, a concrete problem. Teachers may find the question: “what is this useful for?” Irritating. Mathematics is not only an instrumental science and may be practiced just for the quest of knowledge, but this is a legitimate question that deserves an argued answer. Applied mathematics is almost a field on its own and counts as more than half of math production on current research. Of course, one could defend that the whole of physics is applied mathematics, but, to fix definitions, applied maths is the use of non-trivial mathematics on a non-trivial problem, usually requiring the collaboration of experts in both the mathematics and the field in question.

Lamp and 3D printed model displaying the stereographic projection, by Henry Segerman.

Learning about applied mathematics and modelisation is enriching from a cultural point of view, it gives a sense of interconnection between human knowledge, it is also motivating to learn a subject, and unleashes the freedom of mathematics as a language. Formal education includes physics, chemistry, technology, and other sciences, however, there are plenty of examples of modelisation in these fields that make field measures, hundreds of physical constraints, etc. Excluded or neglected by the more classical mathematics curriculum, this field of study is very fruitful for an out-of-school discussion.



The sphere of the Earth, awarded first prize in the Mathematics of Planet Earth exhibit competition 2013.

The initiative Mathematics of Planet Earth [4] focuses on any phenomena related to our planet where mathematics has been a helping or solving tool.

4 Both the name of a research initiative (www.mpe2013.org) as well as a related exhibition managed by IMAGINARY (http://imaginary.org/exhibition/mathematics-of-planet-earth).

Such phenomena can be physical, natural, human, or of any other kind. For instance, cartography is a science that developed from the need of drawing maps of regions of the Earth. As a science, it has multiple branches and concerns; one is how to flatten the curved surface of the Earth into a plane, but also how to measure the Earth (geodesy), instruments for modelling points of view, triggers the problem of representing a spherical surface onto a plane, which is called a map projection. One can think of a multitude of ingenious plane representations of a sphere, drawing the coordinates, projecting imaginary lines on planes, and cylinders... and eventually one needs tools to analyse objectively the different projections, to define and measure distortion, and to evaluate the usefulness of the map for real life problems such as finding routes between locations [5]. Cartography and geodesy were historically a starting point of the mathematical area of differential geometry. Forgetting about some physical constraints is modelling, and developing a theory from this model is abstraction.

Modelisation is also a perfect framework to introduce some areas of mathematics, for instance Differential Equations. While the theory for solving equations with unknown functions and derivatives (both ordinary and partial) is an advanced topic, the idea of a function as modelling a physical magnitude, and constraints involving some change rates of this magnitude is intuitive and accessible. At the Mathematics of Planet Earth there are examples such as models of the movement of ice in a glacier, models of the wind and particles suspended in the atmosphere, models of the sea currents and tsunamis, etc [6].

It is also remarkable that applied mathematics and examples of modelisation suit historical presentations of mathematics perfectly. The history of mathematics (old and recent) is shaped by the domains it relates to. Examples of this historical display are the Winton Gallery at the London Science Museum or the calculus sections at Musée des Arts et Métiers in Paris. Both museums take advantage of their patrimonial collections to show areas where mathematics has played a central role.

4.3. Creativity.

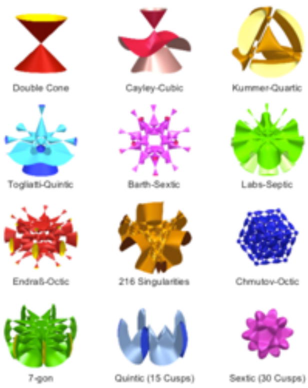
Polydron constructions at MMACA.

A big problem in the formal approach to learning mathematics is the usual lack of freedom to create something, using the language that mathematics provide. Mathematics is presented as a solving tool for predefined problems, and not as a creation tool. While research of new mathematics can be a creative process, research is far away from student and general public experience. Thus, it is important to develop tools and activities that allow the creative expression of the user, using the mathematical language.

Creativity requires an open end of the activity, there is no creativity if the outcome is pre-defined. Modules based on construction blocks provide a traditional space for creativity, especially for the youngest public. Polyhedral

5 See the exhibit The Sphere of the Earth at imaginary.org
6 The Future of Glaciers, Dune Ash, and TsunaMath respectively, at imaginary.org

GeCla, Generation and Classification of plane tilings, by Atractor.



Algebraic surfaces with record of singularities, in SURFER.

building blocks are common in math museums (commercially available [7] or home-made). These blocks allow multiple connections and shapes, but they have also geometric restrictions that make the creation more challenging. Huge, soft blocks are also fun for children, where they can be challenged to build bridges, towers, packings or other structures.

Drawing is another inherently creative activity. Software to draw symmetric tilings, as mentioned before, mixes the creativity of drawing with the "magical" multiplication of the lines drawn, making pleasant patterns. The Atractor association (Portugal) has set inter-school competitions [8] using their software GeCla (Generation and Classification) for plane tilings, where each contestant has to draw a pattern and to guess the symmetry group of other's drawings.

SURFER [9] is an interactive software, part of the IMAGINARY exhibitions, to visualize algebraic surfaces, that is, surfaces in the three-dimensional space defined by apolynomial equation in the three spatial variables x,y,z. The user writes or selects equations as an input, and he obtains pictures of surfaces as an output. The possibilities for the equations are endless, but through a tutorial and experimentation, the user develops an intuition on how changes in the formulae affect the corresponding surface. Ultimately, this is a tool built to use a symbolic, mathematical language in a creative way. SURFER has been used in competitions [10], where participants are asked to create the most beautiful, appealing or interesting image, which is evaluated by a jury.

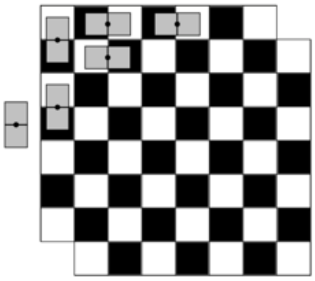
Finally, the maximum exponent of creativity is turning mathematics into a form of art. Mathematical art has an enormous tradition, from the tools of perspective, the golden ratio, or the arabesque geometric tilings, to the art of M. C. Escher, the cubism and modern painters, or geometric sculptures. Today, mathematical artists are a big community, with organizations [11], platforms [12], conferences [13], and competitions [14]. For the public and the learners, the connection between mathematics and art is not only a source of aesthetic satisfaction, but it also gives a compelling feeling that sciences and arts are not opposed fields, but rather all results of creative minds.

4.4. Mind exercise.

Math fairs, exhibitions and museums, tend to have an important part of their collection devoted to games, puzzles and brain teasers. This makes

7 Such as Polydron, Zometool, MathLink...
8 http://www.atractor.pt/mat/GeCla/Competicao-_en.html
9 Available at https://imaginary.org/program/surfer
10 https://imaginary.org/search/node/surfer%20competition
11 e.g. the European Society for the Mathematics and Arts http://www.math-art.eu/
12 e.g. IMAGINARY
13 e.g. Bridges conference http://bridgesmathart.org/
14 e.g. Math Creations, https://imaginary.org/project/math-creations. See also Violet, Wagner, Eremenko "Math Creations - A math-art competition" Proceedings of the Bridges conference 2017.

many of these exhibitions a brain training camp for the visitors, which is by itself a quite positive exercise. Education is not only the transmission of knowledge, it is also training and acquiring competences. Games and riddles help develop strategy and problem solving skills. They also develop reasoning and logical thinking.



The dominoes and mutilated chessboard problem.

A classic game is to find whether it is possible to cover a chessboard with dominoes (yes, it is), then ask whether it is possible if we remove one square of the board (it is not, there is an odd number of squares), and then whether it is possible if we remove two diagonally opposite squares (it is again impossible since there is a different number of black and white squares and each domino covers one of each color). In the first case the solution is affirmative because we can construct a solution. In the second case it is impossible because a necessary condition is not met. In the third case, the previous condition is met but it is still impossible due to another necessary condition (it might be more challenging if the original checkerboard is just a 8x8 grid with no colors). For a public not familiar with rigorous mathematical proofs, such as children (and many adults), this is a challenge to make a logical reasoning to prove a positive or negative result. One can go further asking what is a necessary and sufficient condition for the existence of a solution on a chessboard with removed squares.

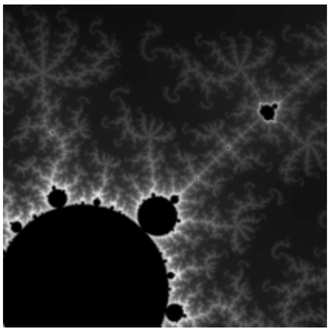
Puzzles can also play a role in the valorization of the knowledge of the visitor. The use of this knowledge from visitors makes them more engaged and satisfied with their position of intellectual advantage. Solving a puzzle is empowering. On any math exhibition, a fascinating phenomenon can be observed: children that manage to solve a puzzle run to tell their parents, brothers, or other children. They feel in possession of a knowledge that is satisfying to share and to be proud of. One of the most famous and difficult puzzle is Rubik's cube. Anyone able to solve it earns a bit of respect for his skill. This is mostly due to the fact that the solution cannot be revealed as a simple trick, you are forced to spend a fair amount of time to learn the solution algorithm, even with a guide to solve it. In the decade of the 80s, with no Internet and Rubik's cube as a novelty, the ultimate skill was to solve the cube. In the next decades, new challenges appeared, such as "speedcubing" to solve it in the least possible time. Today, with world speed records close to the human limit, the attention is shifted to variations of the puzzle, with different structures, challenging the mathematics and the engineering limits. Rubik's cube is a great mathematical toy, it can lead to learning (and researching!) group theory, to develop logic and visual skills, and it is still a children's toy that everyone can play with [15].

The ultimate mind exercise is fostering personal investigation. Not only trying to find a solution to a problem, but discovering the problem itself on the given framework. This process can be surely guided by a mediator, but it is essential to give freedom and time to explore. One of such frameworks

15 See for instance Mathologer's channel on YouTube for mathematical content on the cube. https://www.youtube.com/results?search_query=mathologer+rubik

is the spherical blackboard [16], consisting on a sphere that we can draw on, and possibly a compass and a curved ruler to draw "straight lines" (properly, geodesics). A canvas to draw always sparks creativity, and visitors will try drawing figures, possibly countries resembling the Earth. If faced with the question "can you draw a square?", they will start struggling with the problems of curved space. It is possible to draw a quadrilateral with equal sides, but not right angles. This information does not need to be revealed as a given knowledge, but can be discovered by experimentation. Or rather, it can be guessed after several attempts, but not to be sure without a deeper examination. The fact that there are unlimited possibilities to try forces creativity to make a strategy to understand the problem, and there is always the sense that the experience does not end with the answer to the proposed question. The end goal is not to draw a square. What about triangles, or pentagons? What about drawing a polyhedron over the sphere…?

4.5. Discovering fields.



Piece of the Mandelbrot fractal.

Many fields of knowledge are not present in formal education, just because there is no time to cover everything in school. Many fields of mathematics remain unknown to the general public, even though, in many cases, knowing about their existence is accessible and enriching. Of course, mere introductory notions do not substitute proper courses, but some non-formal approaches serve to discover fields that are unknown or little spread. Furthermore, there are often popular cultural references that can anchor the attention of the public and serve as a starting point for the presentation.

Fractals are one of these popular culture topics on mathematics. Many people have heard about them, maybe they have heard about the Mandelbrot set and seen psychedelic pictures on the Internet. This kind of public is naturally motivated so there is room to push more information into them, while at the same time the learning process is entertaining and pleasant for them.

Fractals link to the notion of infinity, because their construction requires an endless iterative process and a pass to the limit. The self-similarity property is intuitive and it has support on nature, sometimes more evident (trees, fern leaves, types of broccoli), sometimes more revealing (how long is the coast of Great Britain? [17]). The idea of limit of a process can be explored with Peano and Hilbert curves (that fill the plane and space respectively) or von Koch snowflake. Finite approximations can be constructed physically, and computer simulations can be used to explore them virtually [18]. One

16 True chalk blackboards on a sphere exist, but a modern educational set is the Lénárt Sphere (http://lenartsphere.com/), which is transparent.

17 This question is quoted from B. Mandelbrot. The meaning is that there is no good notion of length, since the smaller and smaller details of the abrupt coast we consider, the bigger result we get for the length of the coast, making the length grow to infinity if we use successive smaller units of length. Instead, this problem motivates the idea of fractal dimension.

18 See for instance some Cinderella applets (https://imaginary.org/program/cinderella-applets)

can also explore the notions of fractal dimension, and the iterative process is also the basis to talk about chaos.

Complex numbers and complex dynamics are also essential to understand fractals in depth. Unfortunately, complex numbers are not always a topic granted in non-university education. However, the idea of algebraic operations between points on the plane (or pairs of coordinates, or equivalently complex numbers) is accessible even if the public has no prior knowledge of complex numbers. From this point, one can dive into complex variable functions and complex analysis. The jump is qualitatively huge, and complex analysis is often taught on a second or third year university course in mathematics. However, the idea of the subject can be conveyed to the general public. A complex variable (holomorphic) function happens to be conformal, which means that it preserves angles. This is a notion that can be experienced with the help of appropriate software [19], designed to that purpose. An image (which can be static or can be a video input from a camera) is presented after a holomorphic transformation, and dynamically one can test the angles before and after. One can also see the zeros and poles of meromorphic functions, and play with the complex geometry. The next step is iterating a function (composing it with itself several times) to get to complex dynamics, and from there to Julia sets and Mandelbrot sets.

This journey on complex analysis and fractals needs most probably a mediator with the ability to stop, advance, and skip parts, but in good conditions this is a viable exposition for teenagers and the general public. The ideal setting would be an interactive talk of about 20-25 min in a museum or in an after-school activity, using the software and maybe a few objects (sculpture of a Hilbert curve, a piece of broccoli…). A more classical talk is always an option if the audience is too large, but it is important that the assistants get the opportunity to touch and interact with a holomorphic function or with a fractal, changing parameters and discovering details by themselves.

4.6. Pearls of mathematics.

In mathematics there are endless examples of small stories, maybe a theorem, maybe a conjecture, maybe just some elements of a study field; that can be told without context and can be easily understood, and always contain a surprising result. These stories, that we will call "pearls", are delightful for casual discussions on mathematics, are engaging, and, at the same time, satisfyingly bounded. These pearls look like isolated stories, but every time the story is re-told with more detail, it gets more enriching and connected. For its nature, this plethora of stories does not fit into formal learning, and it has always been perfect for math popularization. Let us see some examples.

The Bannach-Tarski paradox is a theorem, stating that it is possible to cut

19 See the Conformal Webcam by Christian Mercat (https://math.univ-lyon1.fr/~mercat/CindyJS/examples/cindygl/22_webcamconf.html)

a sphere into five pieces, and rearrange them into two spheres, identical in size to the original one. This is called a paradox because it is counter-intuitive and contradicts the additive property of volume, but at the same time it is a proven theorem and it is absolutely true. The explanation has to do with measure theory, and the fact that not all the sets of points that can be defined can also be assigned a volume, they are non-measurable. The pieces on which the spheres need to be cut are so intricate that can't be assigned a volume. But for this discussion, the interesting point is that the story can be told in a few words, and one can appeal to the authority of the sentence "it is a mathematical truth" to astonish the audience and, hopefully, to spark the curiosity and hunger to understand the theorem in more depth [20].

Another pearl is the sum of all natural numbers. The result states that 1 + 2 + 3 + 4 + 5 + … = -1/12. This is an astounding statement and every person will refuse to believe it at first sight. But again, it is mathematically true. The explanation is that the sum is originally only defined for two, three, or any finite number of addends. If we want to make infinite sums, we need to extend the definition of addition, so we can write things like 1 + 1/2 + 1/4 + 1/8 + 1/16 + … = 2. This last addition, or series, is Zeno's paradox and it was astonishing for greeks and for all humanity for centuries, but nowadays it is accepted and used in many areas of mathematics. It relies on a distinction on convergent and divergent series, and the first admit a numerical value while the second don't. In the same way, in some areas like string theory in physics, it may be necessary to assign a numerical value to series that were considered divergent, and in the case of the sum of all natural numbers, the value coherent with the theory is minus one twelfth. This story tells that one can go beyond the rules, and the theory always put to the test. Things discarded or deemed as impossible should always be reconsidered [21].

There are many of such pearls that can be used to surprise and intrigue the public. The format of these stories is usually short books or articles of math popularization and, more recently, on Internet sites such as YouTube. The mathematics popularization video is really a flourishing domain. The relative short duration of the videos (often less than 15 minutes) and the close and casual style has made of this a successful format and the breeding ground for a growing community of math communicators. Language and geographical barriers still exist, but it is a lesser problem for younger generations. Topics are re-formulated between authors with different points of view, often acknowledging a "response", and in different languages for different publics. The audiovisual character of the video allows for mathematical visualizations, animations, diagrams, cartoons, etc, that represent

20 See for instance the video from Vsauce (https://www.youtube.com/watch?v=s86-Z-CbaHA)

21 See videos from Numberphile (https://www.youtube.com/watch?v=w-I6XTVZXww, https://www.youtube.com/watch?v=0Oazb7IWzbA&t=36s), from Mathologer (https://www.youtube.com/watch?v=jcKRGpMiVTw) or from 3brown1blue (https://www.youtube.com/watch?v=sD0NjbwqlYw&t=8s).

a qualitative improvement over text and pictures books. The technique and skills of the authors is, in some cases, as professional as TV productions, without the time and audience constraints of TV.

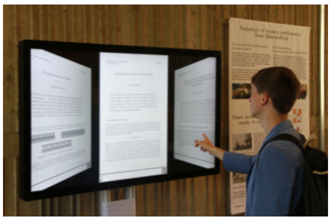
Math communication on short formats, be it on video or the more traditional article, is a huge challenge for didactics. Time, attention, and background constraints push the capacity of synthesis, visual resources, and appeal of the author to the limit. The “pearls” are often the finest introductions to many topics, coated in a mixture of entertainment and mathematical insight. In that regard, the educational value is out of doubt, and these resources should be exploited by teachers and math educators.

4.7. Edge of knowledge.

Mathematics is not a finished science. There are and always will be questions where we don’t know the answer, unsolved problems, and entire domains to be discovered. This is difficult to transmit to the public, since the advances are not apparent in society as it is the case of many technologies (communications, medical research…), nor can be easily described appealing to a clear program as in physics (we discovered a new particle using an accelerator laboratory, because we want to understand matter). Mathematics has for a long time been a rare case where communication of current research was deemed as a futile effort. It is the duty of researchers to publish and communicate their advances to their mathematician colleagues, but it is also their duty, or at least the duty of the research community as a whole, to communicate also with the non-specialists. Fortunately, this tendency has changed in the last decade with many projects around the world. The link between the research and the public can be done by the researchers themselves or by an increasing number of professional math communicators and math outreach organizations.

SURFER on an exhibition.

The Mathematisches Forschungsinstitut Oberwolfach is a research center in Germany. In 2008 it started a math communication project that eventually led to the IMAGINARY organization. Many of the IMAGINARY exhibits are produced by professional mathematicians that visualize and popularize topics of their research. One of these exhibits is the already mentioned SURFER, which visualizes algebraic surfaces. Besides the possibility to explore creative surfaces created by the user, the program gives a tour on the theory of singularities. Singularities are points where an algebraic surface fails to be smooth, such as cusps or edges. These points are rare to find, and hence it is an interesting field of study. For polynomials of low degree (up to 6), there are theorems that find the maximum number of singular points for a surface of the given degree. SURFER provides visualization of these surfaces that are named after the mathematicians that discovered them, in many cases in recent years. For higher degrees (7 and up), there is not yet a definitive result for this problem of finding the maximum possible singularities. These are unsolved problems, and they are currently being researched. Of course, one needs more than SURFER to discover a new theorem, but this exhibit gives a closer sense of what these new theorems would be, and it sparks imagination and the feeling that anyone could write a polynomial that sets a new world record on surface singularities.



“Snapshots of current mathematics”, displayed on an interactive station.

Another project from the same institute, the “Snapshots of current mathematics from Oberwolfach” [22] regularly produces short booklets edited for the general public, that give a glimpse of the topics discussed on Oberwolfach seminars. These booklets are 8 to 10 A5-sized pages long, and can be re-printed and distributed freely in any math communication event, or translated and re-published in general mathematical journals.

Internet platforms for math communication such as Images des Maths [23] (in French) are common meeting point for mathematicians writing. Films and videos are also a tendency in math communication, not only with short format, as already mentioned (such as YouTube), but also full-length films such as Dimensions or Chaos [24] which offer accessible introductions to current research topics in mathematics.

Mathematic communication is already growing from an emergent field to a consolidated domain. More than 50 math museums around the world [25], dozens of conferences [26], and an increasing community of professionals of math communication back this claim. IMAGINARY promotes and provides services to this community (WikiMathCom [27], Math Communication Network [28], IMAGINARY Conferences [29]) and has independently started initiatives, as many Internet channels, which are often supported by national research institutions (CNRS, MSRI...) to encourage math diffusion to the public. International organizations such as IMU and EMS promote actively the mathematical outreach to the general public, with publications, public programs (such as Mathematics of Planet Earth competition), and specific panels on math conferences [30].

The role of math communication as a non-formal mode of learning, and its connection to the formal education provided in schools and universities is still an aspect that is being developed, hoping for a synergy between the two sides of the same coin: to achieve a better education, and a more enlightened society.

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22 Texts on https://imaginary.org/snapshots, see also an interactive exhibition station https://imaginary.org/program/snapshots-slider
23 http://images.math.cnrs.fr/?lang=fr
24 Both by E. Ghys, J. Leys and A. Alvarez http://www.dimensions-math.org/, http://www.chaos-math.org/es
25 http://www.mathcom.wiki/index.php?title=Math_Museums
26 http://www.mathcom.wiki/index.php?title=Conferences
27 http://www.mathcom.wiki
28 http://imaginary.org/network
29 http://ic16.imaginary.org/
30 For instance at the International Congress of Mathematicians ICM (Seoul 2014), https://imaginary.org/event/imaginary-panel-mathematics-communication-for-the-future-a-vision-slam or at the 7th European Congress of Mathematics 7ECM (Berlin 2016) https://imaginary.org/event/ems-session-on-public-awareness-of-mathematics-at-7ecm

5 Inclusive approach

A new vision of museums has been rising as a response to the idea of museums as just a place of conservation. A vision of the museum as cultural haven, inclusive, alive and living, a place that can become meeting points open to the exchange of experiences. This means that the accessibility of contents to an audience as wide as possible assumes a prominent role.

In order to promote accessibility we must first identify what are the fences keeping the audience, or a specific fraction of the audience, from fully enjoying the museum, and work on bringing them down.

This vision is particularly pertinent in relation to interactive science museums. We are, by nature, fully dependant on the active participation of the audience, and our biggest goal to overcome is the transmission of concepts that traditionally depend on specific technical language. To effectively communicate these concepts is part of the core goals of the museum itself.

Therefore, a museum like the Giardino di Archimede that aims from birth to communicate and disseminate mathematics in all its forms and relations to other disciplines, both inside and outside the museum, has an inclusive connotation by nature.

A further discussion on inclusiveness requires taking a long look at the concept of "barrier" in a broad sense: any aspect that may be cause to the exclusion of any specific audience. Many factors are comprised, material and not. These barriers can be overcome, and lead to identifying collectives for whom the enjoyment of the museum is limited or precluded. The reasons for marginalisation can usually be related to a physical or socio-economic disadvantage. In the first group we could find the visually or hearing impaired, the elderly or people with specific disabilities. In the second group we might find foster children, convicts, new immigrants, ethnic minorities with integration difficulties. The final goal of a museum is to open up to as much of a new and wide audience as possible.

Integration and coordination between non-formal education subjects operating in the same area allow for a leap of quality and effectiveness of the proposals. In this sense, the English project "Learning outside Classroom"[1] was very interesting; unfortunately, it was sacrificed to economic choices before being implemented significantly.

In the same line two experiences that have involved the Giardino di Archimede move; they are examples of good practices that could be implemented, even partially, in each country.

The first project aimed at the development of inclusive practices was im-

1 https://www.gov.uk/government/publications/learning-outside-the-classroom
http://www.lotc.org.uk/wp-content/uploads/2011/03/G1.-LOtC-Manifesto.pdf

WELCOME project activities.
Il Giardino d'Archimede.

plemented in 2015. It comprised a network of local museums. It later developed further into a second project named WELCOME (We Encourage Living Collective Open Museums Experiences).

Both projects were born inside a workgroup joined in territory, the city of Florence, but very diverse and heterogeneous in type and topic: art, history and science; public and private. The group was formed initially as a response to a series of initiatives promoted by the Regione Toscana. Despite the differences, the resulting synergy aimed at very concrete common goals that took into account diversity and personal need. The workgroup then constituted into a network called "ArteStoriaScienza - Sistema Coordinato di Musei con Attività di Cooperazione" [2].

The first project realized by the Network was the MuseoBus, attempting to facilitate the insertion of the museums in the living tissue of the city. It pursued an audience that is physically unable to reach the museums autonomously, by weaving a net across neighbouring realities. A series of thematic itineraries were created. These itineraries link each one of the partner museums to a specific local reality [3]. The link was both thematic and physical: a touring bus drove visitors from one point of the itinerary to the next. This first version of the project was, therefore, aimed to an audience that already had a cultural interest, offering them the chance to broaden their interests.

The different itineraries that the Giardino di Archimede organised were:

EXTERNAL INSTITUTION	MATHS INSTITUTION	THEMATIC LINK
Central Dairy	Giardino di Archimede	The geometry of Packaging
Istituto Nazionale di Ottica	Giardino di Archimede	Geometric shapes linked to the reflection of light
Museum	Giardino di Archimede	The geometry of soap bubbles
Mathematics and Computer Sciences	Giardino di Archimede	Liber Abaci and the indo-arabic numbering system.

Among the other different itineraries linked with other museums were visits to a foundry, a textile mill, a pottery workshop, a coffee company, a crystal factory, a nursery maintaining of old plants, the company for public lighting, all connected to a specific museum.

This project contained some trips for specific groups who, for various reasons, might not have the initiative or the ability to plan a visit to the muse-

2 The members of this network are the eight museums in Florence: Fondazione Casa Buonarroti, FirST-Firenze Fondazione Scienza e Tecnica, Museo Galileo, Museo di Storia Naturale dell'Università di Firenze, Museo Fiorentino di Preistoria "P. Graziosi", Museo Marino Marini, Il Giardino di Archimede–Un Museo per la Matematica, Museo Horne.

3 This project had already been started by the rest of the museums in the network. The Giardino di Archimede joined in 2015. This project was cofounded by the Regione Toscana.

um. Both the trips and the activities inside the museums were organized on purpose. Among these groups were some children from a juvenile facility in the suburbs of Florence, some from a Rom community in the metropolitan area, senior centres, rehabilitation centres, institutions for the hearing impaired.

These very positive first experiences encouraged all the partners to develop the project for inclusion in the next WELCOME project, which got public funding and support in 2016, and made a strong point for the accessibility to culture, seeing museums as a place for inclusion and integration. Specifically, the purpose of the project was to increase the usability of the museum by disadvantaged public by implementing various strategies of involvement and strengthening services in favour of visitors. This was attained through the specialization of the creation of dedicated itineraries, choosing new topics and approaches and new types of materials to be used in visits and educational activities.

Activities in the Project [4]

Types of disadvantage	Elderly, disabled, migrants, foster care, patients in hospitals, specific physical or mental disabilities (hearing or visually impaired, autistic, Alzheimer patients)
Actions in the museum	Ad hoc visits, free transportation, design and creation of specific resources and materials.
Actions outside the museum	Preliminary meetings for the preparation of the visits, workshops and activities in the host structures.

In order to identify the specific needs of each group and, therefore, organise the activities accordingly, different existing structures were involved so that promoter museums could get an inside informed view. Collaborations with public institutions and volunteers dedicated to different kinds of social work, both with minors and the elderly, migrants, socially excluded societies, disabled (starting a collaboration with the city paediatric hospital) and convicts [5].

4 The project was submitted by Dr. Cioppi from the Museo di Storia Naturale at the 26th Congress ANMS-Associazione Nazionale Musei Scientifici (Trieste, November, 16-18, 2016) through a communication and a poster and at the demonstration Facciamoci Vedere (Milano, December 13 – 15, 2016). There is also a website for the project (https://welcome-musei-firenze.blogspot.it/), with links to the partner museums.

5 List of the institutions involved in the project: Associazione Anelli Mancanti, Associazione FuoriMercato, Associazione Interculturale Messaggeri di Pace, Associazione Oltre, Caritas - progetto Rom, Casa circondariale di Volterra, Casa circondariale Mario Gozzini, Centri Educativi Gould e Ferretti, Centro Anziani diurno I Tigli, Centro anziani Villa Bracci, Centro di Solidarietà Anconella, Chini Lab, Comunità per minori P. Annibale M. Di Francia, Comunità di Sant'Egidio di Firenze, Cooperativa Il Mandorlo, Cooperativa sociale Le Rose, Cooperativa sociale Matrix onlus, Guidi Raggi RSA, La Cupolina RSA, Le Magnolie RSA, Montedomini RSA, Opera Madonnina del Grappa, Ospedale pediatrico Meyer, Progetto Villa Lorenzi, Residenza Villa Canova, Unione Italiana Ciechi e Ipovedenti, Villa Michelangelo RSA.

For the inclusive approach, the proposal of the museums to the audience has been adapted. A crucial point was choosing the contents and modalities accordingly so that they would be the most interesting and communicative, with the help of experts, and even which resources to use, if necessary, to make them accessible.

USERS	ACTIVITIES	MAIN POINT
Foster children, Rom communities	Little difference	The possibility of a different experience: going to a museum
Teenagers in foster homes	Insertion in the Open Days among the general audience	Working amongst others the same age: origami
Minors at risk of social exclusion	Collaboration with the activity in Day Centres	Specific materials
Rehabilitation Patients	Collaboration with the activity of the centres	
Convicts		
Migrant groups	Guided visits	Intercultural connections
Patients with various mental disorders	Activities in small groups	Personalization of educational interventions
Patients with Alzheimer's	Adapted activities	Stimulus and social insertion
Elders		
Handicapped		
Hospitals	Activities in the centres	

Among the groups that were addressed through the WELCOME project, two had a particularly significant role: the visually and hearing impaired. This was due mostly to the involvement of experts interested precisely in the design of actions and initiatives dedicated to their inclusion. In addition to being the final users of these dedicated proposals, they were also included as co-protagonists of the designing of these proposals.

Activities and exhibits.
Il Giardino d'Archimede.

One of these special collaborations happened with the Unione Italiana Ciechi e Ipovedenti. Their help was fundamental in projecting and designing dedicated resources like captions and guides in braille, with both texts and, eventually, figurines,to make some itineraries in the museums available to the visually impaired.

In the case of interactive museums, like the Giardino di Archimede, some contents are already available to tactile exploration. The itinerary dedicated to the Pythagorean Theorem, for example, consist of wooden puzzles, which are entirely usable by the visually impaired. Captions and instructions for the puzzles were translated in braille, so that total autonomy can be achieved throughout that part of the visit, with or without help from the staff or facilitators. Until now just a part of the itineraries are available to the visually impaired without accompanying persons.

Some interventions can help approaching parts of the exhibitions, for example the 3D printing of different geometrical objects, in scale, so that they can be directly handled for tactile exploration.

The contribution of the Unione Italiana Ciechi e Ipovedenti has been essential in testing the success of these resources, in which some specific properties like the roughness of the finish or the dimensions of these objects could make their interpretation more difficult. Their contribution was important, in general terms, in understanding how to use these old and new resources better. These materials can mostly only work when the museum facilitator can relate to the experience of the visitor.

Some resources like 3D Prints can have multiple and inclusive functions too: they are useful for the visually impaired, they can be an invitation to tactile exploration for any kind of audience, they can reinforce geometrical and special imagination, they can be exhibits to be carried even outside the physical museum.

In order to promote the accessibility to the hearing impaired a collaboration with a Sign Language interpreter was activated. Some museums implemented visits for the hearing impaired with the presence of the SL interpreter.

Some video presentations [6] of the different museums for hearing impaired were produced. In some cases, the videos had captions, in some cases accompanied by an SL translation. These videos were produced with the help of both the interpreter and a video operator who was hearing impaired himself, in the perspective of a participative.

Other resources include audio-visual guides. Again, the idea was to provide a tool that could be useful to the widest audience. The presence of both audio and video allows it use by those with auditory or visual disabilities. In order to overcome the linguistic barrier, we also aim to expand the available languages. Future plans for the inclusion project in the WELCOME network are related to socio-linguistic barriers, addressing linguistic minorities with difficulties integrating in the local cultural scene.

The project also foresees training for the museum personnel aiming to improve the treatment of audiences with special needs.

Training activities were a very important step, considering the specific aims of the project, focusing on a sector of the audience with physical, mental or social difficulties.

In order to assess the success of the project, an evaluation study was commissioned, with interviews and questionnaires for the participants. The cooperation between different kinds of museums, with strong contact points between didactic methodologies and educators and the management result in a tremendous success for the project. Each and all of the museums involved in the project had a mutual, common, solid ground for work, allowing experimentation with various types of audience.

6 https://welcome-musei-firenze.blogspot.com.es/p/sussidi-per-disabili-sensoriali.html

In general, feedback from the visitors in the museums involved in the WELCOME project has been strongly positive, with respect to both the museum personnel (reception, guiding and competence in both subject and communication) and the museums themselves, which were deemed stimulating in both content and interaction. A point of interest shared and widespread across the comments from visitors is connected to novelty: the discovery of something new, never seen or heard before, not yet learnt, new concepts and notions have been the most appreciated point for all kinds of audiences. Novelty is often referred to as the main reason motivating their enthusiasm: museum experience means learning new things.

Other museums with a deep hands-on character, such as the Museum of Mathematics in Catalonia, have had similar experiences (with the visually or hearing impaired, the elderly, groups of children at risk of social exclusion, people with physical and psychological difficulties, convicts, ..), by exploiting the “natural” accessibility of their exhibits or adapting them when needed. The impressions obtained coincide with those collected from Florentine experience.

As already mentioned, the creation of a network of institutions allows a stimulating complicity between specific languages, increases the capillarity of intervention, multiplying both qualitatively and quantitatively its value.V

Our hope is that future implementations and development coherent with the guidelines of a project like WELCOME can transform a visit to a museum into an experience where everyone can feel like a protagonist.

6 A questionnaire to collect some impressions from our users.

6.1. Analysing data

The questionnaires we have distributed in our exhibitions, although diagnostic, have no claim of significance on their own. Indeed, our first goal was to test the validity of the questionnaire itself, its strengths and defects, in order to elaborate a consistent model of internal evaluation that could be proposed and applied more extensively to gain more evidence.

We are aware that a questionnaire is not sufficient to evaluate the whole of the experience. Furthermore, the dimensions of the sample do not allow for a significant analysis.

The diversity of the proposal that each partner member presents to their audience in the project should also be considered.

These questionnaires had to be modified during their testing, to better adapt them to different publics.

For now, we will allow ourselves only generic impressions, capable of starting a reflection, necessary for the design of the Mathspaces exhibition, certainly not dictating conclusions or rules

mathspaces

student evaluation form <8

IDENTIFICATION		
I am:	☐ A boy	☐ A girl
I am from		

EVALUATION	Don't like	Don't know	Yes	I love it!
My motivation before the visit				
I love math	☐	☐	☐	☐
I love games	☐	☐	☐	☐
I love new experiences	☐	☐	☐	☐
I love school	☐	☐	☐	☐
The experience has been				
Funny	☐	☐	☐	☐
Interesting	☐	☐	☐	☐
Easy	☐	☐	☐	☐
Difficult	☐	☐	☐	☐
Different / New	☐	☐	☐	☐
Short	☐	☐	☐	☐
Intense	☐	☐	☐	☐
A waste of time	☐	☐	☐	☐
The staff has been				
Friendly	☐	☐	☐	☐
Tough	☐	☐	☐	☐
Distant	☐	☐	☐	☐

What activity / area did you prefer? ....

AFTER MY VISIT		
I found what I was expecting	☐ Yes	☐ No
Now I like maths more	☐ Yes	☐ No
I would like to return to this museum	☐ Yes	☐ No
I will recomend this museum to my friends	☐ Yes	☐ No

Erasmus+

mathspaces

student evaluation form >8

IDENTIFICATION				
I am:	☐ A boy		☐ A girl	
☐ 9-10 years	☐ 11-12 years	☐ 13-14 years	☐ 15-18 years	☐ > 18 years
I am from				

EVALUATION 0 = totally disagree, 1 = partially disagree, 2 = agree, 3 = totally agree

Evaluate your motivations before the visit				
I love math	0	1	2	3
I love games	0	1	2	3
I love new experiences	0	1	2	3
I love school	0	1	2	3

The experience has been				
Funny	0	1	2	3
Interesting	0	1	2	3
Satisfactory	0	1	2	3
Educational	0	1	2	3

Evaluate the dynamics				
Friendly	0	1	2	3
Lively	0	1	2	3
Collaborative	0	1	2	3
Dispersive	0	1	2	3
Quality of the animator's work	0	1	2	3

mathspaces

student evaluation form >8

Remember…	0 = totally disagree, 1 = partially disagree, 2 = agree, 3 = totally agree			
The activities have been				
Easy	0	1	2	3
Stimulating	0	1	2	3
Different / New	0	1	2	3
Short	0	1	2	3
Intense	0	1	2	3
Sufficient	0	1	2	3
Excessive	0	1	2	3
A waste of time	0	1	2	3

What activity / area did you prefer? ....

What activity / area did you like least? ....

AFTER MY VISIT

I found what I was expecting	☐ Yes	☐ No
I think that my vision of mathematics evolved	☐ Yes	☐ No
I would like to return to this museum	☐ Yes	☐ No
I will recommend this museum to my family / friends	☐ Yes	☐ No

COMMENTS

Erasmus+

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evaluation form _ public

IDENTIFICATION

Gender:

- ☐ Male
- ☐ Female

First contact

- ☐ Yes
- ☐ No

Age group

- ☐ under 19
- ☐ 20-29
- ☐ 30-39
- ☐ 40-49
- ☐ 50-59
- ☐ Over 60

Where are you from?

....

What is the highest degree you completed?

- ☐ Less than Highschool
- ☐ Highschool
- ☐ College/Apprenticeship
- ☐ Bachelor degree
- ☐ Master degree
- ☐ Advanced graduate work

What best describes the organisation you work for:

- ☐ For profit
- ☐ Non-profit
- ☐ Governmental
- ☐ Education
- ☐ Healthcare
- ☐ Other

EVALUATION

0 = totally disagree, 1 = partially disagree, 2 = agree, 3 = totally agree

Evaluate your previous motivations				
I love math	0	1	2	3
I love new experiences	0	1	2	3
I love museums/science centers/exhibitions	0	1	2	3
I love to offer my family/friends new experiences	0	1	2	3

Evaluate the timing				
Short	0	1	2	3
Intense	0	1	2	3
Excessive	0	1	2	3
Sufficient	0	1	2	3

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evaluation form _ public

Evaluate the experience				
Funny	0	1	2	3
Interesting	0	1	2	3
Satisfactory	0	1	2	3
Educational	0	1	2	3
Different / New	0	1	2	3
Easy	0	1	2	3
Stimulating	0	1	2	3
Challenging	0	1	2	3
A waste of time	0	1	2	3

Evaluate the dynamics				
Friendly	0	1	2	3
Lively	0	1	2	3
Collaborative	0	1	2	3
Dispersive	0	1	2	3
Quality of the animator’s work	0	1	2	3

CONCLUSION

I found what I was expecting	☐ Yes	☐ No
I think that my vision of mathematics evolved after my visit	☐ Yes	☐ No
I would like to return to this museum	☐ Yes	☐ No
I will reccomend it to my family / friends	☐ Yes	☐ No

COMMENTS

Erasmus+

mathspaces

evaluation form_teacher

IDENTIFICATION

Gender:	First contact
☐ Male ☐ Female	☐ Yes ☐ No

My degree is in…

☐ Maths ☐ Sciences ☐ ....

I'm teaching

☐ Primary ☐ Secondary ☐ ....

☐ under 8 ☐ 9-13 ☐ 14-16 ☐ 17-18 ☐ Over 18

What subject

☐ Maths ☐ Sciences ☐ ....

I come from

....

EVALUATION 0 = totally disagree, 1 = partially disagree, 2 = agree, 3 = totally agree

Evaluate your previous motivations				
I love math	0	1	2	3
I love games	0	1	2	3
I love new experiences	0	1	2	3
I love museums	0	1	2	3

Evaluate the timing				
Short	0	1	2	3
Intense	0	1	2	3
Excessive	0	1	2	3
Sufficient	0	1	2	3

mathspaces

evaluation form_teacher

Evaluate the experience				
Funny	0	1	2	3
Interesting	0	1	2	3
Satisfactory	0	1	2	3
Educational	0	1	2	3
Easy	0	1	2	3
Stimulating	0	1	2	3
Challenging	0	1	2	3
Different / New	0	1	2	3
A waste of time	0	1	2	3

Evaluate the dynamics				
Friendly	0	1	2	3
Lively	0	1	2	3
Collaborative	0	1	2	3
Dispersive	0	1	2	3
Quality of the animator's work	0	1	2	3

What activity / area did you prefer? ....

What activity / area did you like least? ....

CONCLUSION

I found what I was expecting	☐ Yes	☐ No
I think that the students' vision of mathematics evolved after their visit	☐ Yes	☐ No
I would like to return to this museum	☐ Yes	☐ No
I will reccomend it to my family / friends	☐ Yes	☐ No

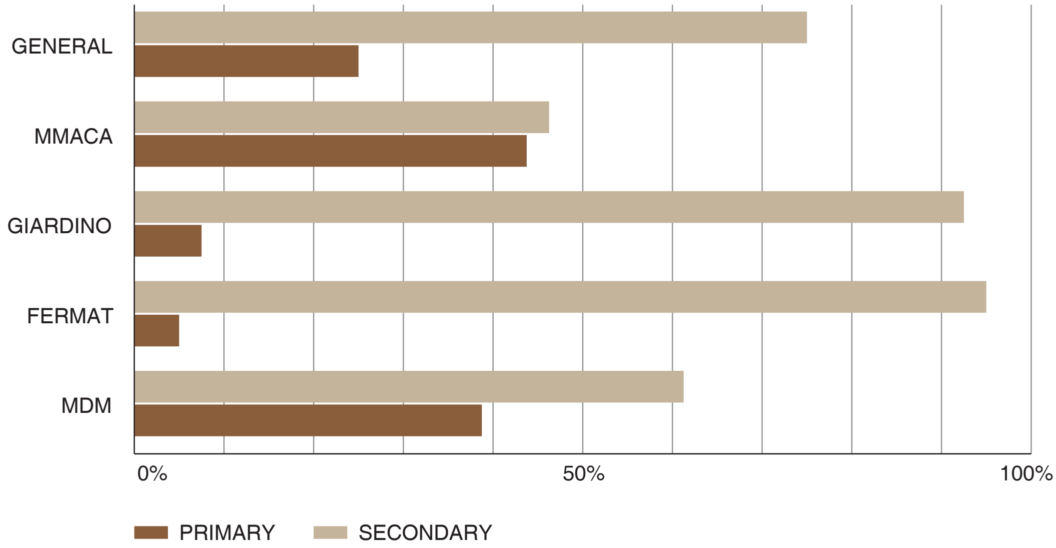
COMMENTS

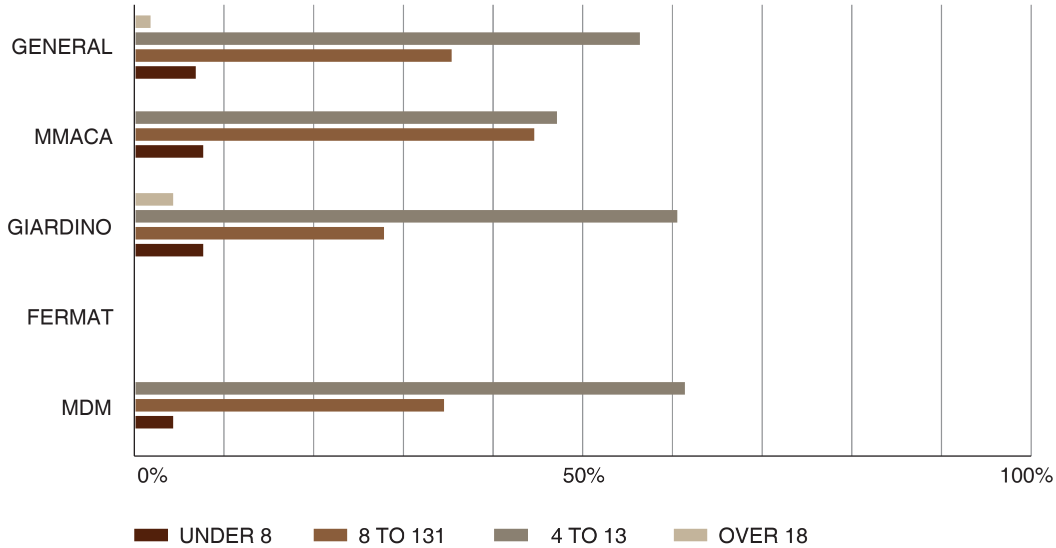
Erasmus+

6.1.1. The composition of the student sample.

We have considered four groups of users: general public, teachers and two groups of students: under and over 8 years - U8 and O8 - considering that around this age a deep change can be made in conceptualizing maths.

(Of course, other key moments are conceivable: introduction of algebra (12-13 years), differential calculus (15-16), which are certainly significant cognitive jumps, but probably have minor effects compared to the forms of the museum proposal).





6.1.2. Student evaluation of the exhibition experience:

U8 group: Experience is considered fun (2 up to 3), interesting (2.4) and new/different (1.75), but with some discrepancy in local situations).

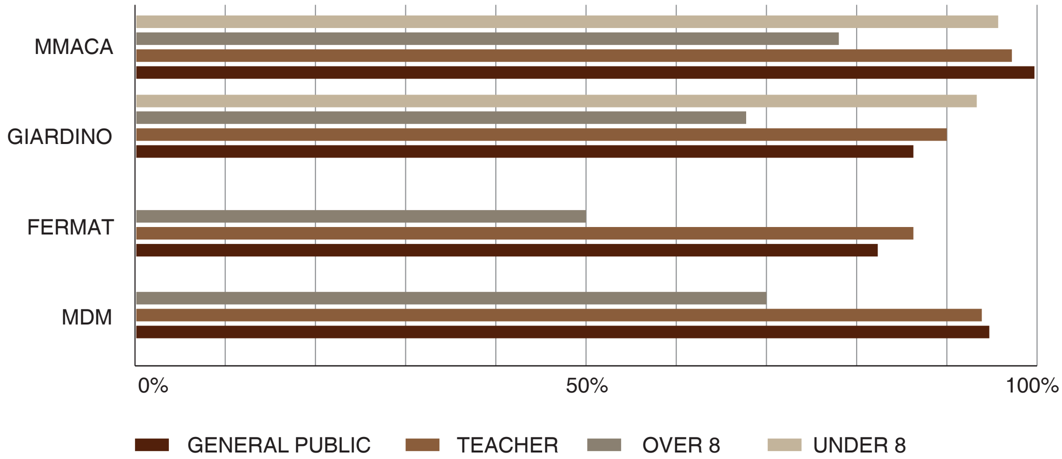
It is not so easy (1.5) and it is certainly not considered a waste of time (0.25).

O8 group: Most answers take about 1.8 (up to 3); so, experience is considered fun, interesting, satisfactory, educational, stimulating and new/different at the same time. It does not look so easy (1.75) and it is certainly not considered a waste of time (0.5).

6.1.3. “I want to return”

U8 group: We have little data, but very indicative: 90% of respondents expressed the desire to return to the museum! It is a similar answer to the adult one, meaning that the educational vocation of our proposal does not limit the offer to a school audience.

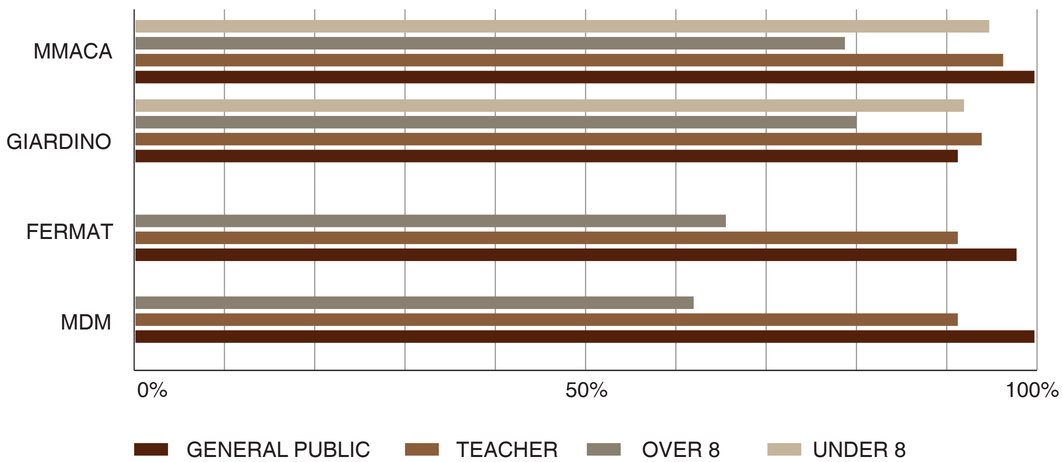
O8 group: The local response ranges from 50 to almost 80% of an intention to return to the museum. Such variability calls for great caution when drawing conclusions and calls for a more in-depth internal analysis of each institution. The group is very composite and includes ages that traditionally give very different answers to the offer of science centres, from the enthusiasm of first adolescence to rejection at sixteen.



6.1.3. “I would recommend”

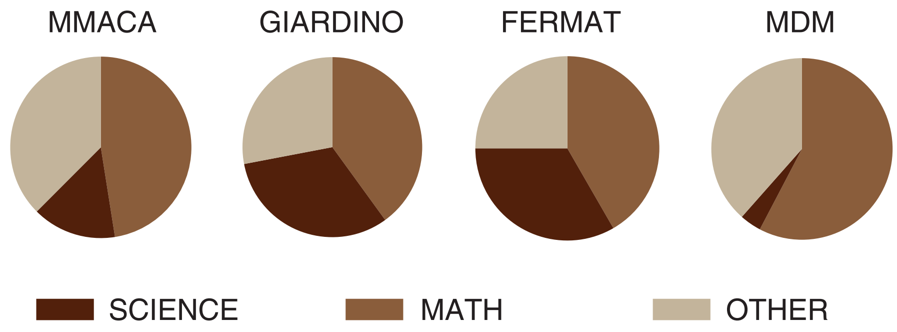
U8 group: We have little data, but very indicative: 98% of respondents expressed they would very much recommend a visit to our exhibitions, showing once again a full agreement with the adult audience.

O8 group: The answer is less enthusiastic (and more varied), but about 70% would recommend a visit to our exhibitions.

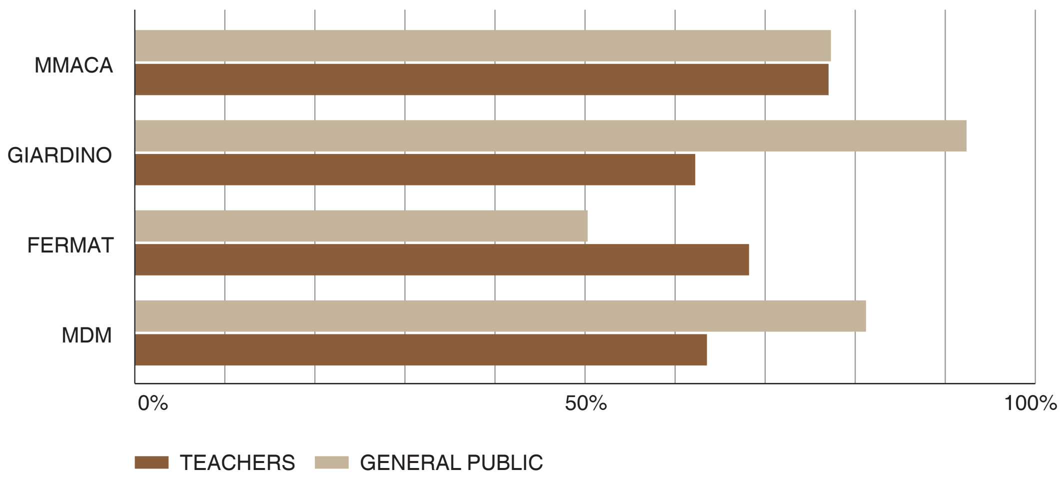


6.2. What teachers come to our exhibition?

As it was logical to expect, mathematics teachers are almost half of those who accompany their pupils to our visits, despite the fact that local champions have a different composition between primary and secondary school teachers.



Depending on the situation, most part of users was coming to the museum for the first time (from 50% to 90%).

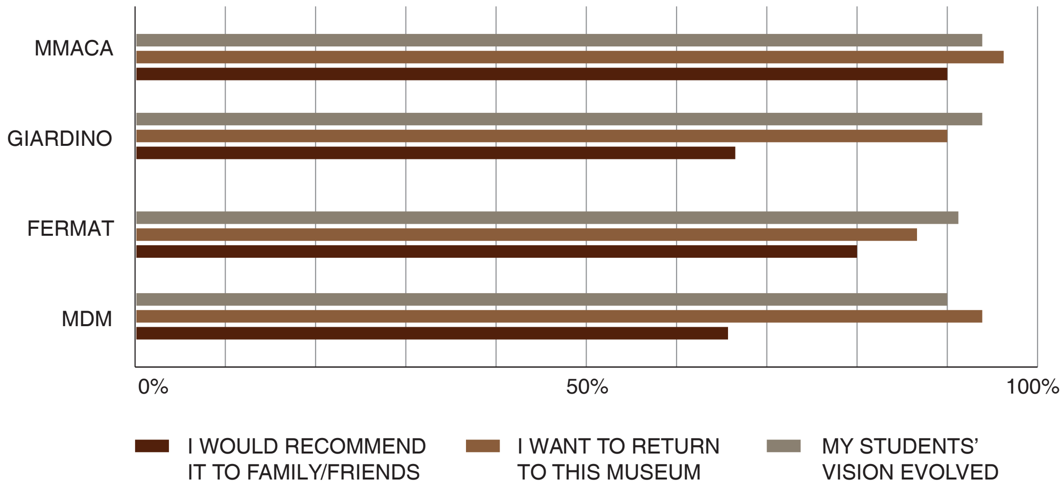


The factors involved are many and different from each other: extension of the catchment area; antiquity of the institution; convenience of transport; targeted school/museum projects...

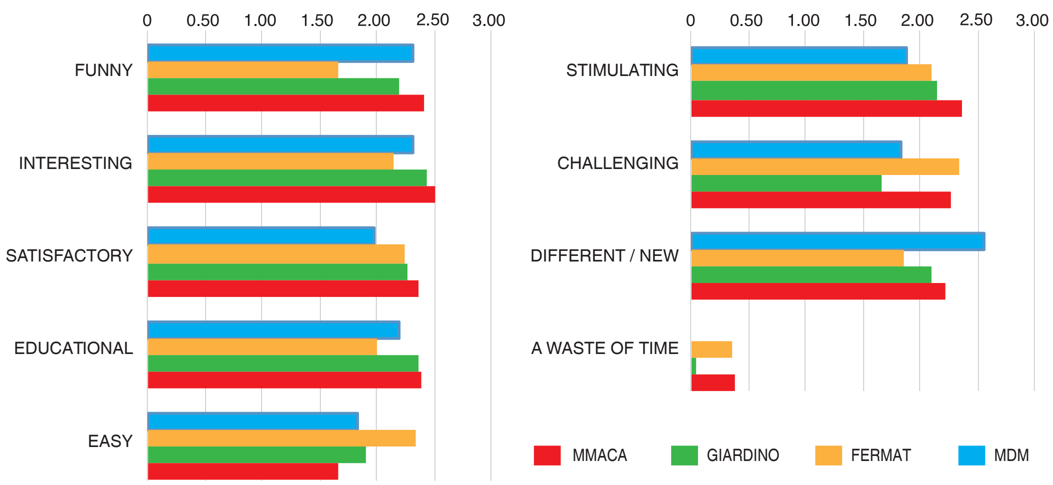
However, the data suggests elements of reflection: how to expand our area of influence, the need to train teachers so that the visit is more productive and linked to class work, etc.

6.3. What do teachers think of our exhibition?

Calling once again for prudence, given the small sample, the results of the survey are absolutely rewarding. The visit provides a new view of mathematics to 65-90% of teachers who express their intention to return (85-95%) and to recommend the visit (90-95%)



The success of our proposal is confirmed by the judgment given by teachers: interesting (2.75 out of 3), satisfactory (2.5), educational (2.5), stimulating (2.5), new / different (2.2).



Interestingly, the differences that appear in judgment of difficulty of content and activities: seem smaller (2.7) in a sample almost entirely formed by secondary school teachers and, on the contrary, it increases (1.8) when users are younger.

And, once again, the visit is everything but a waste of time (0.4)!

6.4. The role of educators.

Finally, an important role in the positive judgment of all partners' proposals, despite the different way of conducting activities, seems to be due to cultural educators-facilitators-mediators (so many names for so many functions!). In all realities, their work is rated around 2.5 out of 3 and for all publics (here also with a slight fall in the teenager sample).